Roll No. | | | | | | | | | | | | |  Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (Software Engineering) (Sem.–1)

# LINEAR ALGEBRA FOR ENGINEERS

Subject Code : MA-1300

M.Code : 77256

Time : 3 Hrs. Max. Marks : 60

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

## SECTION-A

**1. Solve the following :**

a) Find the general solution of the linear system whose augmented matrix is $\begin{bmatrix} 1 & -3 & -5 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix}$.

b) Reduce the matrix $\begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{pmatrix}$ to row echelon form.

c) Find the inverse of the matrix $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.

d) Examine whether the transformation $T : R^2 \to R^2$ defined as $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} |x| \\ -y \end{pmatrix}$ is linear or not?

e) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and let $k$ be a scalar. Find a formula that relates det $kA$ to $k$ and det $A$.

f) Let $a = \begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}$ and $b = \begin{bmatrix} -7 \\ -4 \\ 6 \end{bmatrix}$. Compute $||a + b||^2$.

g) Show that similar matrices have same eigen values.

h) If $\lambda$ is an eigen value of A, show that $\lambda^{-1}$ is an eigen value of $A^{-1}$.

i) Check whether the vectors $u = \begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$ are orthogonal or not?

j) The characteristic roots of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & k & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 0, 3, 15. Find the value of $k$.

## SECTION-B

2. a) Determine if the following system is consistent :

$$y - 4z = 8$$

$$2x - 3y + 2z = 1$$

$$4x - 8y + 12z = 1$$

b) Let $u = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$, $v = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$ and $w = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$, For what value(s) of $h$ is $w$ in the plane spanned by $u$ and $v$ ?

3. a) Given $A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ 2 \\ 9 \end{pmatrix}$, write the augmented matrix for the linear system that corresponds to the matrix equation $Ax = b$. Then solve the system and write the solution as a vector.

b) Find the inverse of the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$ using row transformations.

4. Let $T : R^3 \to R^3$ be a linear transformation defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ x+y \\ x+y+z \end{pmatrix}$. Find the matrix representation of $T$ *w.r.t.* the ordered basis $B_1 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $B_2 = \{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}$.

5. a) Let $v_1 = (1, -1, 0)$, $v_2 = (0, 1, -1)$ and $v_3 = (0, 0, 1)$ be elements of $R^3$. Show that the set of vectors $\{v_1, v_2, v_3\}$ is linearly independent.

b) Prove that $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$, where $w$ is a cube root of unity.

## SECTION-C

6. a) Let $u_1 = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$, $u_2 = \begin{bmatrix} -4 \\ -5 \\ 8 \end{bmatrix}$, and $v = \begin{bmatrix} 8 \\ 2 \\ -9 \end{bmatrix}$. Determine whether $v$ is in the subspace of $R^3$ generated by $u_1$ and $u_2$.

b) Solve the following system of linear equations by Cramer's rule :

$x + y + z = 6,\ x - y + 2z = 5,\ 3x + y + z = 8$

7. Determine the eigen values and corresponding eigen vectors of the matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

8. Diagonalize the matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

9. Find an orthogonal basis or the coloumn space of the matrix $\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & 3 \\ 3 & -7 & 8 \end{bmatrix}$



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