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Unit - I: SEQUENCE AND SERIES

1. State and prove squeeze lemma.
2. State and prove Comparison test, Root test, Ratio test.
3. Convergent sequences are bounded.
4. a. If (sn) converges to s and (tn) converges to t, then (sn+tn) converges to s+t

   b. If (sn) converges to s and (tn) converges to t, then (sn.tn) converges to st

   c. If (sn) converges to s ,if sn≠0 for all n and if s≠0 then $(\frac{1}{sn})$ converges to $\frac{1}{s}$

   d. If (sn) converges to s and (tn) converges to t, if sn≠0 for all n and if s≠0 then $(\frac{tn}{sn})$ converges to $\frac{t}{s}$
5. A sequence is a convergent sequence iff it is a Cauchy sequence. (or)

   Every Cauchy sequence of real numbers is convergent
6. Convergent sequences are Cauchy sequences.
7. All bounded monotone sequences converge.
8. Every sequence (sn) has a monotone subsequence.
9. State and prove Bolzano – Weierstrass theorem (or)

   Every bounded sequence has a convergent subsequence
10. A series converges iff it satisfies the Cauchy criterion.
11. Problems on comparison, root, ratio and alternating series theorem.
12. If sequence (sn) converges to a positive real number s and (tn) is any sequence, then lim sup(sntn)=s lim sup tn.
13. If the sequence (sn) converges, then every subsequence converges to the same limit.
14. Prove that the following

    a. $\lim_{n\to\infty}\frac{1}{n^p} = 0 \, for \, p > 0$

b. $\lim_{n\to\infty} a^n$=o if lal<1

c. $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$

d. $\lim_{n\to\infty} a^{\frac{1}{n}} = 1 \; for \; a > 0$

15. Let $t_1 = 1$and $t_{n+1} = \frac{{t_n}^2+2}{2t_n}$ $for\ n \geq 1, then$ assume (t_n) converges and find the limit.

16. Cauchy sequences are bounded.(or)

Every Cauchy sequence of real numbers is bounded.

17. Every convergent sequence is bounded. Is converse true? Give example

18. Prove that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ convergent if p>1 by using integral test.

Unit - II: CONTINUITY

19. Define continuous function, uniformly continuous function.

20. Let f be a continuous real valued function on a closed interval [a, b]. Then f is a bounded function.

21. State and prove intermediate value theorem.

22. If f is continuous on a closed interval [a , b], then f is uniformly continuous On [a , b].

23. If f is uniformly continuous on a set S and (s_n) is a Cauchy sequence in S, Then (f (s_n)) is a Cauchy sequence.

24. If f is uniformly continuous on a bounded set S , then f is a bounded function on S.

25. Problems on uniform continuity.

26. Let f(x) =x^2 sin($\frac{1}{x}$) for x≠0 and f(0) = 0. Prove f is continuous at 0.

27. Suppose f is a real valued continuous function on R and f(a)f(b) < 0 for Some a,b in R. Prove there exists x between a and b such that f(x)= 0.

28. Let f be a real valued function with dom (f) €R. Then f is continuous atx_0 if and only if for every monotone sequence $(x_n$) in dom(f) converging to x_0 , we have lim f(x_n)=f(x_0)

29. Prove x =cos x for some x in (0 ,$\frac{\Pi}{2}$).

30. If f is uniformly continuous on its domain[a ,b] then show that f is continuous on

Its domain [a,b].

Unit - III: DIFFERENTIATION

31. Sate and prove Rolle's theorem, Mean value theorem and Taylor's theorem.

32. Suppose that f is differentiable at a then prove

a. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(a)$

b. $\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h} = f'(a)$

33. Prove that | cosx-cosy|≤|x-y|

34. Prove that if 'f' is differentiable at 'a' , then 'f' is continuous at 'a'

35. a. Find the Taylor series for cosx for all x

b. Find the Taylor series for sinx for all x

36. Define derivative of a function 'f' at a point 'a'

37. Find the following limits if they exists

a.$\lim_{x\to 0}\frac{x-\sin x}{x}$

b. .$\lim_{x\to 0^+}\frac{1+\cos x}{e^x-1}$

c. .$\lim_{x\to 0}\frac{1-cox2x-2x^2}{x^2}$

d.$\lim_{x\to\infty} x^{sin\frac{1}{x}}$.

38. Use the definition of derivative to calculate the derivatives of the following functions at the indicated points

a. x^3 at x=2

b. g(x)=x+2 at x=a

c. f (x)=$x^2\ cosx\ at\ x = 0$

d. r(x)=$\frac{3x+4}{2x-1}$ at x=1

39. Prove that $\lim_{x\to 0}\frac{cosx-1}{x^2} = \frac{1}{2}$.

40. Prove that $\lim_{x\to 0}\frac{logx}{x}= -\infty$.

41. a. Show that x<tanx for all x€[0,$\frac{\Pi}{2}$]

b. Show that $\frac{x}{sinx}$ is a strictly increasing function on (0,$\frac{\Pi}{2}$)

c. Show that x ≤ $\frac{\Pi}{2}$ sinx for x€[0,$\frac{\Pi}{2}$]

42. Find the Taylor series for sin hx=$\frac{1}{2}(e^{x}$-$e^{-x})$ and cox hx=$\frac{1}{2}(e^{x}$+$e^{-x})$

43. Prove that $\lim_{x\to\infty}(1-\frac{1}{x})^{x}=\frac{1}{e}$.

44. Prove that if f and g are differentiable on R, if f(0)=g(0) and if f'(x)≤g'(x) for all x€R , then f(x)≤g(x) for x≥0

45. Find the following limits if they exists

a.$\lim_{x\to 0}\frac{x^{3}}{sinx-x}$

b. .$\lim_{x\to 0^{+}}\frac{tanx-x}{x^{3}}$

c. $\lim_{x\to 0}[\frac{1}{sinx}-\frac{1}{x}]$

d.$\lim_{x\to\infty}(cosx)^{\frac{1}{x}}$

Unit-4 INTEGRATION

46. Define lower darboux sum, upper darboux sum, lower darboux integral, upper darboux integral and darboux integral. [Hint:Darboux is also known as Riemann]

47. Let f be a bounded function on [a , b]. if P and Q are partitions of [a , b] , Then L(f , P)≤ U(f , Q).

48. Let f be a bounded function on [a, b], then L (f)≤ U (f).

49. A bounded function f on [a , b] is integrable iff for each there exists a Partition P of [a , b] such that U(f , P) – L(f , P) <£ .

50. Let f be a bounded function on [a, b]. if P and Q are partitions of [a , b] and P contain in Q , then L(f , P)≤ L(f , Q)≤ U(f , Q)≤ U(f , P).

51. State and prove Cauchy criterion for integrability.

52. A bounded function f on [a , b]is Riemann integrable iff it is a darboux Integrable.

53. a. Every monotone function f on [a , b] is integrable.

b. Every continuous function f on [a , b] is integrable.

c. Every constant function is integrable.

54. State and prove fundamental theorem of calculus.

55. State and prove intermediate value theorem for integrals.

56. If f and g are integrable on [a , b], then prove that $\int_a^b f \leq \int_a^b g$, for all x in [a, b].

57. If f is integrable on [a , b], then |f| is integrable on [a , b] and $|\int_a^b f| \leq |\int_a^b f\ |$.

58. Show $|\int_{-2\Pi}^{2\Pi} x^2 sin^8(e^x)dx| \leq \frac{16\Pi^3}{3}$.

59. Let f be a function defined on [a , b]. if a < c < b and f is integrable on [a , c]

and on [c , b], then prove that

(a). f is integrable on [a , b] and

(b). $\int_a^b f = \int_a^c f + \int_c^b f$

60. Let f(x) = {x for rational/0 for irrational ,then calculate the upper and lower darboux integrals for f on the intervals [a , b]

All the best