Subject Title: Algebra Prepared by: S Shravani
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Unit - I

1. A group G, identity element is unique.
2. In a group G, inverse element is unique.
3. Prove that the set Z of all integers form an abelian group w.r.t the operations defined by a*b = a+b+2, for all a,b∈ Z
4. Cancellation laws holds in a group. (let G be a group. Then for a,b, c∈ G, ab=ac ⇒ b = c and ba=ca⇒ b = c.
5. In a group G for a,b,x,y∈ G the equation ax = b and ya = b have unique solutions.
6. If every element of a group (G,.) is its own inverse , show that (G,.) is abelian group.
7. The order of every element of a finite group is finite and less than or equal to the order of a group.
8. In a group G ifa∈ G, then $|a|= |a^{-1}|$.
9. If a is an element of group G such that |a| = n, then a^n=eiff n/m.
10. If a is an element of group G such that |a| = 7, then show that a is the cube of some element of G.
11. A non-empty complex H of a group G is subgroup pf G iff (i) a∈ H, b ∈ H ⇒ ab ∈ H
    a. (ii) a∈ H,a^{-1} ∈ H.
12. A non-empty complex H of a group G is subgroup pf G iffa ∈ H, b ∈ H ⇒ ab^{-1} ∈ H
13. The necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G is a∈ H, b ∈ H ⇒ ab ∈ H.
14. If H and K are two subgroups of a group G, then HK is subgroup of G iff HK = KH
15. If H_1 and H_2 are two subgroups of a group G, then $H_1 \cap H_2$is also a subgroup of G.
16. The union of two subgroups of a group G is a subgroup iff one is contained in other.
17. Every cyclic abelian group is an abelian group. Converse is not true.
18. Every subgroup of cyclic group is cylic.
19. If a cyclic group G is generated by an element of order n, the a^n is generator of G iff (m,n) = 1.
20. The order of a cyclic group is equal to its generator.
21. Show that the group (G= {1,2,3,4,5,6} ,$\times_7$) is cyclic . also write down all its generators.

22. How many subgroups does Z_{20} have? List them.

23. If G is an infinite cyclic group , the G has exactly two generators which are inverse of each other.

24. Write down the following products as disjoint cycles.
    (i) (1 3 2) (5 6 7) (2 6 1) (4 5)
    (ii) (ii) (1 3 6) (1 3 5 7) (6 7) (1 2 3 4).

25. Definations:
    i. Group
    ii. Subgroup
    iii. Addition modulo
    iv. Multiplication modulo
    v. Cyclic group
    vi. Permutation group
    vii. Order of a group
    viii. Order of an element

Unit -III

26. If R is a Boolean ring then (i) a+a=0 for all a ∈ R (ii) a+b = 0 ⇒ a=b and **(**iii) R is commutative under multiplication.
27. Find all units of Z_{14}

28. The intersection of two subring of a ring R is a subring of R, A ring has no zero divisors iff the cancellation laws holds in R, A field has no zero-divisors.
29. List all zero-divisors in z_{20} Can you see a relationship between the zero-divisors of z_{20} and the units ofz_{20} ?, A field is an integral domain.
30. Every finite integral dimain is a field.
    The characteristic of an integral domain is either a prime or zero.
31. A field has no proper nin-trivial ideals.
    A commutative ring R with unity element is a field if R have no proper ideals.
    The union of two ideals of a Ring R is a ideal iff one is contained in other.
    Every ideal of z is a principal ideal.
32. An ideal U of a commutative ring R with unity is maximal iff the quotient eing R/U is a field.
    Find all maximal ideals in a. Z_8. b. Z_{10}. c. $Z1_2$. d. Z_n.
33. 1. Definations:

    Ring, Boolean ring, Sub ring
    Integral domain
    Field
    Zero divisor
    Characteristic of a ring
    Idempotent and nilopotent
    Ideal
    Principal ideal
    Maximal ideal
    Factor ring
    Prime ideals

Unit IV

34. 1. Test by divisibility by 9.
    2. Prove that a ring homomorphism carries an idempotent to an idempotent.
    3. The homomorphic image of a ring is a ring.
35. Let R, R' be two rings and ϕ : R → R' be a homomorphism. Foe every ideal U' in a ring R' , f^{-1} (U') is an ideal in R.
36. Let f(x) = $4x^3 + 2x^2 + x + 3$ and g(x) = $3x^4 + 3x^3 + 3x^2 + x + 4$, where f(x), g(x) ∈ $Z_5[x]$.
    Compute f(x) + g(x) and f(x).g(x).

37. If ϕ is a homomorphism of a ring R into a ring R' the ker ϕ is an ideal of R.
If ϕ is a homomorphism of a ring R into a ring R' then R' then ϕ is an into isomophism iff
Every quotient ring of a ring is a homomorphic image of a ring.
Fundamental theorem of homomorphism

38. The division algorithm.
Factor theorem.
Determine all ring homomorphisms from Z to Z.
39. Definitions:
    i. Homomorphism ring
    ii. Isomorphism ring
    iii. Homomorphic image of ring
    iv. Monomorphism ring
    v. Automorphism ring
    vi. Kernel of a homomorphism of ring
    vii. Polynomial ring
    viii. Degree of polynomial

Unit II

40. Any infinite cylic group is isomorphic to integers Z.
Cayley's theorem.
Find the regular permutations group isomorphic to the multiplicative group $\{1, -1, i, -i\}$.
41. Let (G , .) , (G' , .) be two groups. let ϕ be a homomorphism from G into G' then (i) $\phi(e) = e'$ where e is the identity in G and e' is identity in G'. (ii) $\phi(a^{-1}) = \{\phi(a)\}^{-1}$.
42. Every homomorphic image of an abelian group is abelian.
commute.
43. let ϕ be a isomorphism from G onto G' then G = (a)iff G' = (ϕ(a)) .

44. let ϕ be a isomorphism from G onto G' then for any elements a and b in G, a and b commute iff ϕ(a) and ϕ(b)
45. show that the mapping $\phi : G \to G$ such that $\phi(a) = a^{-1}$ for all a ∈ G, is an automorphismof a group G iff G is abelian.
The set of all automorphism of a group G forms a group w.r.t composition of mapping.
46. The set of all inner automorphism of a group G forms a group w.r.t composition of mapping.
H is any subgroup of a group (G , .) and h ∈ G then h∈ H, iff hH=H=Hh.
If a and b are any two elements of group G and H is subgroup of group G then
47. Any two left (right) cosets of a subgroup are either disjoint or identical.
If H is a subgroup of a group G fora,b∈ G the relation a ≡ b(modH) is an equivalence relation.
If H is a subgroup of a group G then there is one-one correspondence between the set of all distinct left cosets of H in G and the set of all distinct right cosets of H in G.
48. Lagrange's theorem.
If G is a finite group and a ∈ G , then |a|/|G|.
If p is a prime number then every group of order p is cyclic group i.e a group of prime order is cyclic.
Orbit – Stabilizer theorem.
49. A subgroup of H of a group G is normal , if xHx^{-1} = H for all x ∈ G.
A subgroup of H of a group G is a normal subgroup of G iff each left coset of H in G is a right coset of H in G.
The set A_n of all even permutations on n symbols is a normal subgroup of the permutation group S_n on the n symbols.
50. A subgroup of H of a group G is a normal subgroup of G iff product of two left(right) cosets of H in G is again left(right) coset of H in G.
Every subgroup of an abelian group is normal.

If G is a group and H is a subgroup of index 2 in G, then H is normal subgroup of G.
The union of two normal subgroups of a group G is a normal subgroup.
H is normal subgroup of G. the set ^{G} of all cosets of H in G w.r.t coset multiplication is a group

51. 1. Definitions:
   i. Homomorphism Group
   ii. Isomorphism Group
   iii. Homomorphic image of a Group
   iv. Monomorphism Group
   v. Automorphism Group
   vi. Coset
   vii. Congruence modulo
   viii. Index
   ix. Normal subgroup
   x. Factor group
   xi. Kernel of a homomorphism