BSc Odd Semester Question Bank

Paper-1: Differential Calculus

UNIT-I

1. If y = $\frac{\log x}{x}$, show that $\frac{d^2y}{dx^2} = \frac{2\log x-3}{x^3}$

2. If y =$\frac{a+bx}{c+dx}$ then show that $2y_1y_3 = 3y_2^2$.

3. If $ax^2 + 2hxy + by^2 = 1$, show that $\frac{d^2y}{dx^2} = \frac{h^2-ab}{(hx+by)^3}$

4. Find the n[th] derivative of (i) y = $\frac{x^2}{(x+2)(2x+3)}$ (ii) y = $\frac{x^4}{(x-1)(x-2)}$ (iii) y = $\frac{1}{a^2-x^2}$

5. If y = Sin ax + Cos ax, prove that y_n=$a^n\sqrt{[1-(-1)^n Sin\, 2ax]}$

6. State and prove Leibnitz theorem.

7. If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$

8. If y = $Sin(msin^{-1}x)$, prove that $(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n$ and also find $y_n(0)$.

9. State and prove Maclaurin's theorem.

10. Using Maclaurin's theorem expand (i) Sin x (ii) Tan x (iii)log(1+sinx) (iv) e^xsec x

11. State and prove Taylor's theorem.

12. Expand log(sin x) in powers of (x-2) by Taylors theorem.

13. State and prove (i) Rolle's (ii) Lagrange's (iii) Cauchy Mean Value theorems.

14. Find 'c' by Lagrange's theorem for f(x)=$\sqrt{x^2-4}$, a=2 and b=3.

15. Verify Cauchy MVT for f(x)=e^x and g(x)=e^{-x} between x $\in [a,b]$

UNIT-II

1. Determine the limits (i) $\lim_{x\to 0}\frac{a^x-1-xloga}{x^2}$ (ii) $\lim_{x\to 0}\frac{\log(1-x^2)}{\log\cos x}$ (iii)$\lim_{x\to 0}\frac{a^x-b^x}{b}$

2. Determine the limit of $\frac{\log(x-a)}{\log(e^x-e^a)}\ as\ x \to a$

3. Determine (i) $\lim_{x\to 0}[\frac{1}{x^2}-\frac{1}{sin^2x}]$ (ii)) $\lim_{x\to 0}[\frac{1}{x^2}-Cot^2x]$

4. Determine (i) $\lim_{x\to 0}[\cos x]^{1/x^2}$ (ii) $\lim_{x\to 0}[\cos x]^{\cot x}$ (iii) $\lim_{x\to a}[2-x/a]^{Tan\,\pi x/2a}$

5. Define curvature, radius of curvature, total curvature, length of an arc as a function.

6. Find the radius of curvature in different forms.

7. For a cycloid $x=a(t+sint), y=a(1-cost)$, prove that $\rho = 4\,\text{acos}\left(\frac{1}{2}\right)t$

8. Show that the curvature at the point (3a/2,3a/2) on the curve

$x^3+y^3=3axy\ is\ \frac{-8\sqrt{2}}{3a}$.

9. In the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that the radius of curvature at an end of the major axis is equal to the semi-latus rectum of the ellipse.

10. Show that the radius of curvature at any point of the astroid $x=acos^3\theta$, $y=asin^3\theta$ is equal to twice the length of the perpendicular from the origin to the tangent.

11. Define circle of curvature, chord of curvature, centre of curvature, evolute, involute.

12. Prove that the evolute of the hyperbola $2xy=a^2\ is\ (x+y)^{2/3}-(x-y)^{\frac{2}{3}}=2a^{2/3}$

13. Show that the circle of curvature at the point $(am^2, 2am)$ of the parabola y^2=4ax as its equation as $x^2+y^2-6am^2x-4ax+4am^3y=3a^2m^4$

UNIT-III

1. Define Partial Differential equation, homogeneous function

2. If $u=\ x^2Tan^{-1}\left(\frac{y}{x}\right)-y^2Tan^{-1}\left(\frac{x}{y}\right), where\ xy\neq 0$, prove that $\frac{\partial^2u}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$

3. If $u=\ e^{xyz}$, show that $\frac{\partial^3u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)e^{xyz}$

4. If $u=Log(x^3+y^3+z^3-3xyz)$, show that

(a) $[\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}]^2u=\frac{-9}{(x+y+z)^2}$ (ii) $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}=\frac{-3}{(x+y+z)^2}$

5. State and prove Euler's theorem for homogeneous functions.

6. Corollary of Euler's theorem.

7. If $u = Tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ $where\ x \neq y,$ then show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = Sin\ 2u$ (ii) $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = (1 - 4sin^2 u)sin2u$

8. Verify Euler's theorem for $z = ax^2 + 2hxy + by^2$.

9. If H = (y −z, z −x, x − y) prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$

10. If $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a, show\ that\ \frac{d^2y}{dx^2} = \frac{a}{(1-x^2)^{3/2}}$

11. Obtain Taylor's formula for the function e^{x+y} at (0,0) for n=3.

12. Expand the function $f(x,y)=x^2+xy-y^2$ by Taylor's theorem in powers of (x-1) and (y+2).

UNIT-IV

1. Define maximum and minimum value, stationary value, extreme value.

2. Find the extreme values of $5x^6+18x^5+15x^4-10$.

3. Show that the maximum and minimum values of (x+1)(x+4)(x-1)(x-4) are -9 and -1/9 respectively.

4. Show that the maximum value of $x^{1/x}$ is $e^{1/e}$.

5. Using Lagrange's condition discuss the maximum and minimum values of u where $u = (x^3y^2)(1-x-y)$.

6. show that the minimum value of $u = xy + (a^3/x)+(a^3/y)$ is $3a^2$.

7. Lagrange's method of undetermined multipliers.

8. Find the max and min of $x^2+y^2+z^2$ subject to $ax^2+by^2+cz^2=1$.

9. Find the minimum value of $x^2+y^2+z^2$, given that ax+by+cz=p.

10. In a plane triangle find the maximum value of u = Cos A CosB CosC

11. Find the asymptotes parallel to the coordinate axes for the curves

$(x^2+y^2)x- ay^2$ and $x^2y^2-a^2$.

12. Find the asymptotes of $x^3+2xy-xy^2-2y^3+xy-y^2-1=0$.

13. Find the asymptotes of $x^3-x^2y-xy^2+y^3+2x^2-4y^2+2xy+x+y+1=0$.

14. Find the envelope of a straight line x cosα + ysinα =l sinα cosα, for α being parameter.

15. Find the envelope of the family of curves $axsec\alpha - bycosec\alpha = a^2 - b^2$