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Subject Title: Integral Calculus | Prepared by: Ms Afreen Begum

Year: 3 | Semester: 5 | Updated on: 30.12.2020

Unit - I: AREAS AND VOLUMES

1. Definition of Double integrals, Riemann sum, Integral over a Rectangle, Properties of the integrals
2. Integrate (i) $\int^{2}\int^{3}(x^2 + y)dy\,dx$ (ii) $\int^{\pi/2}\int^{1} e^x cosy\,dx\,dy$
3. Compute $\int_0^2\int_{y^2}^4 y\cos x^2 dx\,dy$, by changing the order of integration
4. State Fubini's theorem.
5. Change the order of integration and evaluate $\int_0^1\int_0^x sinxdydx + \int_1^2\int_0^{2-x} sinx\,dydx$
6. Let R=[-3,3]x[-2,2]. Without explicitly evaluate $\iint_R(x^5 + 2y)$
7. Find the area of the region ,using double integrals, bounded by y=2-x^2 and x-y=0,2x+y=0
8. Integrate the function f(x, y)=3xy over the region bounded by y=32x^3 and y=$\sqrt{x}$.
9. Evaluate $\iint_D (x - 2y)dA,$ where D is the region bounded by y=x^2+2 and y=2x^2-2.
10. Use double integrals to find the area of the region bounded by the parabola y=2-x2, and the lines x-y=0, 2x+y=0.
11. Find the volume of the region under the graph of f(x,y)=2-IxI-IyI and above the xy-plane .
12. Change of order of integration and evaluate (i)$\int_0^1\int_0^x(2 - x - y)dydx$
(ii)$\int^2\int^{4-2x} y\ dy\ dx$ (iii)$\int^2\int^{4-y^2} x\ dx\ dy$.
13. Find the volume of the region bounded by a graph of f (x,y)=$2x^2 + y^4\sin\pi x$ on top ,the xy-plane on the bottom and the planes x=0,x=1,y=-1,y=2 on the sides.
14. Compute $\int_{-1}^5\int_{-1}^2(5 - |y|)dxdy$

15. Discuss the types of elementary regions in the plane used in evaluating the double integrals.

Unit-II TRIPLE INTEGRALS

16. Define Integral over a box, Riemann sum of triple integrals, triple integrals, Fubini's theorem of triple integrals, Elementary region in space, types of triple integrals.
17. Find the volume inside the capsule bounded by the paraboloids $z=9-x^2-y^2$ and $z=3x^2+3y^2-16$.
18. Evaluate the triple integrals (i) $\iiint_{[-1,1]x[0,2]x[1,3]} xyz\, dV$ (ii) $\iiint_{[1,e]x[1,e]x[1,e]} \frac{1}{xyz} dv$.
19. Evaluate $\int_0^1 \int_{1+y}^{2y} \int_z^{y+z} z\, dx\, dz\, dy$.
20. Integrate the given function over the indicated region W

    (i) $f(x,y,z)=2x-y+z$, W is the region bounded by the cylinder $z=y^2$, the xy-plane and the planes $x=0$, $x=1$, $y=-2$, $y=2$.

    (ii) $f(x,y,z)= 8xyz$; W is the region bounded by the cylinder $y=x^2$, plane $y+z=9$ and the xy- plane.
21. Change the order of integration of $\int_0^1 \int_0^1 \int_0^{x^2} f(x,y,z) dz\, dx dy$ to give the five other equivalent iterated integrals.
22. Find the volume of the solid bounded by $z=4-x^2$, $x+y=2$ and the coordinate planes.
23. Find the volume of the solid bounded by the planes $y=0$, $z=0$, $2y+z=6$ and the cylinder $x^2+y^2=9$.
24. Find the volume of the solid bounded by the paraboloid $z=4x^2+y^2$ and the cylinder $y^2+z=2$.
25. Find the volume of the solid over the function $f(x,y,z)=4x+y$ and W is the region bounded by $x= y^2$, $y=z$, $x=y$ and $z=0$.
26. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
27. Evaluate $\int_1^3 \int_0^z \int_1^{xz} (x+2y+z) dxdydz$.
28. Integrate the function $f(x,y,z)=x+y$ over the region bounded by $x^2+3z^2=9$ and $y=0$, $x+y=3$.
29. Compute $\iint_W 3x dv$ where W is the regionin the first octant bounded by $z=x^2+y^2$, $x=0$, $y=0$ and

z=4.

30. Evaluate $\iiint_B f(x,y,z)dv$ where B is the tetrahedron with the vertices (0,0,0),(1,0,0),(0,1,0) and (0,0,1) and f(x,y,z)=1+xy.

Unit-III CHANGE OF VARIABLES

31. Change of variables and coordinate transformation, Jacobian of double and triple integrals.
32. Evaluate $\iint_D (x^2 - y^2)e^{xy}dxdy$, where D is the region in the first quadrant bounded by the hyperbolas xy=1, xy=4 and the lines y=x, y= x+2.
33. Double integrals in polar coordinates, Cartesian coordinates, general coordinates
34. Change of variables in triple integrals, triple integrals in cylindrical coordinates, spherical coordinates.
35. Calculate the volume of the cone of height 'h' and radius 'a', in which the cone is a solid W bounded by the surface $az = h\sqrt{x^2 + y^2}$ and the plane z=h. (using both cylindrical and spherical coordinates).
36. If T(u,v)=(3u,-v),find the matrix A such that T(u,v)=A$\begin{pmatrix} U \\ V \end{pmatrix}$.
37. Suppose T(u,v)=$\left[\frac{u+v}{\sqrt{2}}, \frac{u-v}{\sqrt{2}}\right]$,describe how T transforms the unit square [0,1]x[0,1]?
38. If T(u,v,w)=(3u-v,u-v+2w,5u+3v-w),describe how T transforms the unit cube [0,1]x[0,1]x[0,1]?
39. Evaluate the integral $\int_0^1 \int_{\frac{y}{2}}^{\frac{y}{2}+2} (2x - y)dxdy$ by using substitutions u=2x-y , v=y.
40. Determine the value of $\iiint_W (x + y + z)dv$ where W denotes the solid region in the first octant between the sphere $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$ where 0<a<b.
41. If T(u,v)=$\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$ and D^* is the parallelogram whose vertices are (0,0),(1,3),(-1,2) and (0,5) . Determine D=T(D^*).
42. Find the area of the region inside both of the circle r=2a cosθ and r=2a sinθ where a is a positive constant.
43. Find the volume of a ball of radius a using spherical coordinates.
44. Find the volume of a cone of radius a and height h using spherical coordinates.
45. Evaluate $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^{3} \frac{e^z}{\sqrt{x^2+y^2}} dzdydx$ by using cylindrical coordinates.