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Subject Title: Vector calculus

Prepared by: B.Lalitha

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Unit - I: LINE INTEGRALS AND SURFACE INTEGRALS

1. Define line integral.

2. Define Surface integral.

3. If $F=xyi-zj+x^2k$ and C is the curve $x=t^2$, $y=2t$, $z=t^3$ from t=0 to t=1.Evaluate $\int_c F.dr$.

4. If $F=(3x^2+6y)-14zj+20xzk$ then evaluate the line integral $\int_c F.dr$ from (0,0,0) to (1,1,1) along x=t, y=t ,z= t^3 .

5. If $F=x^2 y^2 i+yj$ then evaluate $\int_c F.dr$ where C is the curve $y^2=4x$ in the XY plane from (0,0) to (4,4).

6. Prove that the work done by a force F depends on the end points and not on the path in a conservative field.

7. Find the line integral $\oint_c$ rxdr where the curve C is the ellipse $x^2/a^2+y^2/b^2=1$ taken in anti clock wise direction .what do you notice about the magnitude if the answer?

8. If $F=(5xy-6x^2)i+(2y-4z)j$ Evaluate $\int_c F.dr$ along the curve c in the xy-plane given by y=x3 from the point (1,1) to (2,8).

9. Compute the line integral $\int$(y2 dx-x2dy)around the triangle whose vertices are (1,0),(0,1) and (-1,0).

10. Find the line integral of F= (y,-x,0) along the curve consisting of the two st.line segments

    a) $X=1, 1\le y\le 2$ b) $y=1, 0\le x\le 1$

11. Evaluate $\iint_s A.n\,ds$ where $A=18zi-12j+3yk$ and S is that of the plane $2x+3y+6z=12$ which is located in the first octant.

12. Defined work done by force

Unit - II: VOLUME INTEGRALS,GRADIENT,DIVERGENCE AND CURL.

13. Define Volume integral.

14. If $z=f(x+ay)+\emptyset(x-ay)$, prove that $\partial^2z/\partial y^2=a^2\partial^2z/\partial x^2$

15. Define Gradient.

16. Define Divergence.

17. Define Curl.

18. Compute the gradient of the scalar function f(x,y.z) = e^{xy} (x+y+z) at (2,1,1).

19. Find a unit normal vector to the surface $x^2 + y^2 + 2z^2 = 26$ at the point(2,2,3).

20. Find the unit normal to xy=z2 at(1,1,-1).

21. Find the angle between the two surfaces $x^2+y^2+z^2=9$, $x^2+y^2-z=3$ at (2,-1,2).

22. Find the directional derivative of $2xy+z^2$ at (1,-1,3) in the direction of i+2j+3k.

23. Find the volume of the tetrahedron with vertices at (0,0,0),(a,0,0),(0,b,0) and (0,0,c).

24. If $F=(2x^2-3z)i-3xyj-4xk$, evaluate ∇.F dv and ∇xF dv where v is the closed region bounded bt x=0,y=0,z=0,2x+2y+z=4.

25. Show that the vector field $F=(x^2+xy^2)i+(y^2+x^2y)j$ is the conservative and find the scalar potential function.

Unit - III: DIVERGENCE AND CURL OF A VECTOR FIELD

26. If A is a vector function find div(curlA).

27. If $f=x^3 i+y^3 j$+z3k then find div curl F.

28. Show that the vector e^{x+y-2z} (i+j+k) is solenoidal.

29. Prove that F=yz+zx+yxk is irrotational

30. Find the value of a,b.c such that the following vector is irrotational F= (x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k.

31. If F is a conservative vector field show that curl F=0

32. Find divF, where $F=r^n$ r. find n if it is solenoidal.

33. Evaluate $\nabla^2 \log r$ where $r=\sqrt{(x^2+y^2+z^{2)}}$.

34. Show that ∇. (∇. F) = ∇X(∇Xf) + ∇^2 f

35. Show that the vector field $F=(x^2-yz)I+(y^2-zx)j+(z^2-xy)k$ is conservative and find the scalar potential function corresponding to it.

36. Find the curl f=grad (x3 +y3 +z3 -3xyz).