Unit - I: Differential Equations of first order and first degree

1. Find the general solution of $\sqrt{1-x^{2}} \mathrm{~d} y+\sqrt{1-y^{2}} \mathrm{dx}=0$
2. Solve the differential equation $\left(e^{x}+1\right) y \mathrm{~d} y+(y+1) \mathrm{d} x=0$
3. Solve : $x d x+y d y=\frac{x d x+y d y}{x^{2}+y^{2}}$
4. Solve $\frac{d x}{Z^{2} y}=\frac{d y}{Z^{2} X}=\frac{d z}{y^{2} X}$
5. Solve: $\frac{d x}{x^{2}-y^{2}-z^{2}}=\frac{d y}{2 x y}=\frac{d z}{2 x z}$
6. Solve : $\frac{d y}{d x}=\frac{y^{2}+y+1}{x^{2}+x+1}$
7. Solve : $\left(x^{2}+y^{2}\right) \mathrm{d} y=2 x y d x$
8. Solve $\left(1+e^{\frac{x}{y}}\right) \mathrm{dx}+e^{\frac{x}{y}}\left[1-\frac{x}{y}\right] \mathrm{dy}=0$
9. Solve : $(x+y-1) d y=(x+y+1) d x$
10. Solve $\left(1-x^{2}\right) \frac{d y}{d x}+2 x y=x \sqrt{1-x^{2}}$
11. Solve $(x+y+1) \frac{d y}{d x}=1$
12. Show that the necessary and sufficient condition for the differential equation $M d x+N d y=0$ to be exact is that $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.
13. Solve $y \sin 2 x d x-\left(1+y^{2}+\cos 2_{x}\right) d y=0$
14. If the differential equation $M d x+N d y=0$ is homogeneous and $M x+N y \neq 0$, then show that $\frac{1}{M x+N y}$ is the integrating factor .Solve $x^{2} y \mathrm{dx}-\left(x^{3}+y^{3}\right) \mathrm{dy}=0$
15. Solve: $\left(3 x^{2} y^{3} e^{y}+y^{3}+y^{2}\right) \mathrm{dx}+\left(x^{3} y^{3} e^{y}-x y\right) d y=0$
16. Solve $\left(y^{2}+2 x^{2} y\right) d x+\left(2 x^{3}-x y\right) d y=0$
17. Solve $\left(x y^{2}-x^{2}\right) d x+\left(3 x^{2} y^{2}+x^{2} y-2 x^{3}+y^{2}\right) d y=0$
18. Solve $\mathrm{xdx}+\mathrm{ydy}=a^{2}\left[\frac{x d y-y d x}{x^{2}+y^{2}}\right]$
19. Solve $(y z+2 x) d x+(z x-2 z) d y+(x y-2 y) d z=0$
20. Solve : $\left(y^{2}+y z\right) d x+\left(z^{2}+z x\right) d y+\left(y^{2}-x y\right) d z=0$
21. Solve $\frac{x d x}{y^{2} z}=\frac{d y}{x z}=\frac{d z}{y^{2}}$
22. Solve $\frac{d x}{x\left(y^{2}-z^{2}\right)}=\frac{d y}{y\left(z^{2}-x^{2}\right)}=\frac{d z}{z\left(x^{2}-y^{2}\right)}$

UNIT-2
Differential Equations of first order but not of first degree and applications of first order differential equations
23. Solve $x^{2} p^{2}+x y p-6 y^{2}=0$
24. Solve $p=\log (p x-y)$
25. The charcoal from a tree killed in the volcanic eruption that formed a lake contained $44.5 \%$ of $14_{C}$ found in living matter .About how old is the lake (half life of $14_{C}$ is 5600 approximately).
26. Find the Orthogonal trajectories for $\mathrm{y}=c_{1} e^{-x}$
27. Solve $(p-x y)\left(p-x^{2}\right)\left(p-y^{2}\right)=0$
28. Solve $p^{2}+2 \mathrm{py} \cot \mathrm{x}=y^{2}$.
29. Solve $(p+y+x)(x p+y+x)(p+2 x)=0$
30. Solve $p^{3}(x+2 y)+3 p^{2}(x+y)+(y+2 x) p=0$
31. Solve $\mathrm{y}+\mathrm{px}=\mathrm{p}^{2} x^{4}$ (or) Solve $\mathrm{y}+\mathrm{px}=x^{4} p^{2} \quad\left[\mathrm{p}=\frac{d y}{d x}\right]$
32. Solve $\mathrm{y}=2 \mathrm{px}+\tan ^{-1}\left(x \mathrm{p}^{2}\right)$ where $\mathrm{p}=\frac{d y}{d x}$
33. Solve $y=y p^{2}+2 p x$.
34. Reduce $(y-p x)(p-1)=p$ to clairaut's form and find the solution.
35. Solve $\sin \mathrm{px} \cos \mathrm{y}=\cos \mathrm{px} \sin \mathrm{y}+\mathrm{p} \quad$ where $\mathrm{p}=\frac{d y}{d x}$
36. Solve $y=2 p x+y^{2} p^{3}$.
37. The bacteria in a colony can grow unchecked by the law of exponential growth $\mathrm{y}=y_{0} \mathrm{e}^{k t}$. The colony starts with one bacterium and doubles every half hour. How many bacteria will the colony contain at the end of 24 hours .
38. If 100 mg of radium is reduced to 90 mg of a radium in 200 years. Determine how much radium will remain at the end of 1000 years.Also find the half life of radium.
39. It is found that 0.5 percent of radium disappears in 12 years.
(a)What percentage will disappear in 1000 years
(b) What is the half life of radium?
40. If Rs. 10,000 is invested at $6 \%$ per annum,find what amount has accumulated after 6 years if interest is compounded (a)Annually (b)Quarterly (c)Continuously.
41. Find the family orthogonal to the family $y=c e^{-x}$ of exponential curves.Determine the members of each family passing through $(0,4)$.
42. Find the orthogonal trajectory of $r=c_{1}(1-\sin \theta)$.

