

Roll No. 

Total No. of Pages : 02

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B.Tech. (IT) (2018 Batch) (Sem.-3)

**MATHEMATICS-III**

Subject Code : BTAM-304-18

M.Code : 76393

Time : 3 Hrs.

Max. Marks : 60

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

**SECTION-A**
**Write brief :**

1. Express  $\partial w / \partial r$  and  $\partial w / \partial s$  in terms of  $r$  and  $s$  if

$$w = x + 2y + z^2, \quad x = \frac{r}{s}, \quad y = r^2 + \ln s, \quad z = 2r$$

2. Show that the function

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

is not continuous at origin.

3. Find the local extreme values of the function  $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$
4. Define convergence of a sequence and give an example of a convergent sequence.
5. State Leibniz's test for alternating series.
6. Determine for what values of  $a$  and  $b$ , the following differential equation is exact.

$$(y + x^3)dx + (ax + by^3)dy = 0$$

7. Find the integrating factor for the following differential equation

$$(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$$

8. Give an example of a fourth order linear differential equation.
9. Find the solution of the differential equation  $y'' - 4y' - 12y = 0$
10. If  $y_1$  and  $y_2$  are two linearly independent solutions of a second order linear differential equation, then what can you say about the general solution of this equation. Justify your answer.

### SECTION-B

11. Find the volume of the region in the first octant bounded by the coordinate planes and the planes  $x + z = 1$  and  $y + 2z = 2$ .
12. For what values of  $x$  does the following power series converge ?

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$$

13. Obtain the general solution and singular solution of the non-linear equation

$$y = xy' - (y')^3.$$

14. Solve the differential equation  $y'' + 16y = 32 \sec 2x$  by using method of variation of parameters.
15. If  $\sum_{n=1}^{\infty} |a_n|$  converges, then show that  $\sum_{n=1}^{\infty} a_n$  also converges. Is the converse also true? Justify your answer.

### SECTION-C

16. Find the extreme values of the function  $f(x, y, z) = x^2 + y^2 + z^2$  subject to the constraints  $x^2 + y^2 - 1 = 0$  and  $x + y + z = 1$ .
17. Test the convergence of the series (i)  $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$  (ii)  $\sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{1 + n^2}$
18. Find the general solution of the equation  $y'' - 4y' + 13y = 18e^{2x} \sin 3x$ .

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**