Roll No. Total No. of Pages : 02

Total No. of Questions : 18

B.Tech.(CE) (2018 Batch) (Sem.–3)

MATHEMATICS-III (TRANSFORM & DISCRETE MATHEMATICS)

Subject Code : BTAM-301-18

M.Code : 76373

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

SECTION-A

Write briefly :

1. Define gradient of a scalar point function.
2. If $F = (x + y + 1)\, i + j - (x + y)\, k$. Show that $F.\ curl\ F = 0$
3. Define Laplace transform.
4. Write the relation between Laplace and Fourier transform.
5. Represent $f(t) = \sin 2t,\ 2\pi < t < 4\pi$ and 0 otherwise, in terms of unit step function.
6. Define Solenoidal and irrotational fields.
7. State convolution theorem of Fourier transform.
8. State Stokes theorem.
9. Write Euler's formula of Fourier series.
10. Write Gibbs phenomenon.

SECTION-B

11. Find the values of a and b such that the surfaces $ax^2 - byz = (a + 2)\,x$ and $4x^2\,y + z^3 = 4$ cut orthogonally at (1, –1, 2).

12. Apply Convolution theorem to evaluate the inverse Laplace transform of :

$$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$

13. Find the Fourier sine transform of $e^{-|x|}$. Hence show that

$$\int_0^\infty \frac{x \sin mx}{1 + x^2}\,dx = \frac{\pi e^{-m}}{2}, \quad m > 0$$

14. Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary of the area enclosed by the x-axis and the upper-half of the circle $x^2 + y^2 = a^2$.

15. If A and B are irrotational, prove that A × B is solenoidal.

SECTION-C

16. Verify Gauss divergence theorem for $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le x \le c$.

17. Find the Fourier cosine series of the function $f(x) = \pi - x$ in $0 < x < \pi$. Hence show that

$$\sum_{r=0}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}$$

18. a) Use Laplace transform method to solve :

$$\frac{d^2x}{dt^2} - \frac{2dx}{dt} + x = e^t$$

With $x = 2$, $\frac{dx}{dt} = -1$ at $t = 0$.

b) Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). Also calculate the magnitude of the maximum directional derivatives.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.