Topic:- MATHS MPHIL S2

1) Which of the following societies awards the Fields Medal?

[Question ID = 10202]

1. International Mathematical Union

   [Option ID = 40802]
2. American Mathematical Society

   [Option ID = 40803]
3. European Mathematical Society

   [Option ID = 40804]
4. International Mathematical Society for Outstanding Research

   [Option ID = 40806]

Correct Answer :-

- International Mathematical Union

  [Option ID = 40802]

2) Which of the following mathematicians won the famous Abel Prize for 2019 on the work of analysis, geometry and mathematical physics?

[Question ID = 10205]

1. Robert Langlands

   [Option ID = 40813]
2. Karen Uhlenbeck

   [Option ID = 40814]
3. Louis Nirenberg

   [Option ID = 40816]
4. S.R. Srinivasa Varadhan

   [Option ID = 40817]

Correct Answer :-

- Karen Uhlenbeck

  [Option ID = 40814]

3) Which of the following Journals/Magazines/Newsletters is published in India

[Question ID = 10206]

1. The Mathematics Student

   [Option ID = 40818]
2. The College Mathematics Journal

   [Option ID = 40820]
3. Mathematics Magazine

   [Option ID = 40821]
4. Involve

   [Option ID = 40823]

Correct Answer :-

- The Mathematics Student

  [Option ID = 40818]

4) Which of the following mathematicians is not a recipient of Padma Bhushan, the third highest civilian honor in India?

[Question ID = 10209]
1. Manjul Bhargava

[Option ID = 40828]
2. M.S. Narasimhan

[Option ID = 40830]
3. S.R. Srinivasa Vardhan

[Option ID = 40832]
4. Akshay Venkatesh

[Option ID = 40833]

Correct Answer :-
- Akshay Venkatesh

[Option ID = 40833]

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5) Which mathematician of the post-christian era wrote the mathematical treatise 'Ganita Sara Sangraha'?

[Question ID = 10210]
1. Mahavira

[Option ID = 40834]
2. Sridharacharya

[Option ID = 40835]
3. Brahmagupta

[Option ID = 40836]
4. Varahamihira

[Option ID = 40837]

Correct Answer :-
- Mahavira

[Option ID = 40834]

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6) Let T be the set of square free positive integers i.e. the set of positive integers not divisible by any square larger than 1. Assuming that any positive integer can be written as $n = m^2 k, k \in T, m \in \mathbb{N}$, the value of $\sum_{k \in T} \frac{1}{k^2}$ is equal to

[Question ID = 10211]
1. $\frac{\pi^2}{16}$

[Option ID = 40838]
2. $\frac{\pi^2}{6}$

[Option ID = 40839]
3. $\frac{16}{\pi^2}$

[Option ID = 40840]
4. $\frac{15}{\pi^2}$

[Option ID = 40841]

Correct Answer :-
- $\frac{15}{\pi^2}$

[Option ID = 40841]

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7) Which of the following statements is true?

[Question ID = 10212]
1. $f(x) = \frac{1}{x-2}$ is uniformly continuous on $(2,3]$.

[Option ID = 40842]
2. $f(x) = x^2$ is uniformly continuous on $[2,+\infty)$.

[Option ID = 40843]

3. $f(x) = \sin\left(\frac{1}{x}\right)$ is uniformly continuous on $(0,1)$

[Option ID = 40844]

4. $f(x) = e^{-x^2}$ is uniformly continuous on $(-\infty, +\infty)$.

[Option ID = 40845]

Correct Answer :-

- $f(x) = e^{-x^2}$ is uniformly continuous on $(-\infty, +\infty)$.

[Option ID = 40845]

8) Applying term wise differentiation or otherwise, the value of $\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$ is

[Question ID = 10213]

1. $6e^2$

[Option ID = 40846]

2. $4e^2$

[Option ID = 40847]

3. e^2

[Option ID = 40848]

4. $6e$

[Option ID = 40849]

Correct Answer :-

- $6e^2$

[Option ID = 40846]

9) Which of the following statements is true?

[Question ID = 10214]

1. If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and the series $\sum_{n=1}^{\infty} b_n$ diverges, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

[Option ID = 40850]

2. If $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} |a_n| = B$, A and B are finite then $|A| = B$.

[Option ID = 40851]

3. If the series $\sum_{n=1}^{\infty} a_n$ diverges and $a_n > 0$, then the series $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ also diverges.

[Option ID = 40852]

4. The series $\sum_{n=1}^{\infty} \frac{n+1}{n+5}$ converges to $\frac{1}{5}$.

[Option ID = 40853]

Correct Answer :-

- If the series $\sum_{n=1}^{\infty} a_n$ diverges and $a_n > 0$, then the series $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ also diverges.

[Option ID = 40852]

10) The value of $\int_C (2xy - x^2)dx + (x + y^2)dy,$ where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$ is

[Question ID = 10215]

1. $\frac{1}{30}$

[Option ID = 40854]

2. $\frac{1}{25}$

[Option ID = 40855]

3. $\frac{2}{35}$

[Option ID = 40856]

4. $\frac{3}{17}$

[Option ID = 40857]

Answer:
30
[Option ID = 40854]

11) Let $A(2,5)$ be rotated by an angle of $\frac{\pi}{3}$ then the coordinates of the resulting point is

[Question ID = 10216]

1. $\left(1+\frac{5\sqrt{3}}{2}, \sqrt{3}-\frac{5}{2}\right)$

[Option ID = 40858]

2. $\left(1-\frac{5\sqrt{3}}{2}, \sqrt{3}-\frac{5}{2}\right)$

[Option ID = 40859]

3. $\left(1-\frac{5\sqrt{3}}{2}, \sqrt{3}+\frac{5}{2}\right)$

[Option ID = 40860]

4. $(1+\frac{5\sqrt{3}}{2}, \sqrt{3}+\frac{5}{2})$

[Option ID = 40861]

Correct Answer :-

- $(1+\frac{5\sqrt{3}}{2}, \sqrt{3}+\frac{5}{2})$

[Option ID = 40861]

12) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a length preserving linear transformation. Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the matrix corresponding to T. Which of the following relations is false?

[Question ID = 10217]

1. $a^2 + c^2 = 1$

[Option ID = 40862]

2. $b^2 + d^2 = 1$

[Option ID = 40863]

3. $ab + cd = 0$

[Option ID = 40864]

4. $ab + cd = 1$

[Option ID = 40865]

Correct Answer :-

- $ab + cd = 1$

[Option ID = 40865]

13) Let $T : V \to W$ be a nonsingular linear transformation of a finite dimensional vector space V to any vector space W. Which of the following is true?

[Question ID = 10218]

1. dim V < Rank of T

[Option ID = 40866]

2. dim V = Rank of T

[Option ID = 40867]

3. T maps a basis of V to a basis of W

[Option ID = 40868]

4. dim V > Rank of T

[Option ID = 40869]

Correct Answer :-

- dim V = Rank of T

[Option ID = 40867]

Let $A = \begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix}$ be a matrix over a field F of characteristic 0. Which of the following statements is true?

[Question ID = 10219]

1. If $F = \mathbb{R}$ then A has nonzero eigenvalues.

   [Option ID = 40870]
2. If $F = \mathbb{R}$ then A is diagonalizable

   [Option ID = 40871]
3. If $F = \mathbb{C}$ then A has real eigenvalues

   [Option ID = 40872]
4. If $F = \mathbb{C}$ then A is diagonalizable

   [Option ID = 40873]

Correct Answer :-

- If $F = \mathbb{C}$ then A is diagonalizable

  [Option ID = 40873]

15) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map defined by
$T(x,y) = (2x-7y, 4x+3y)$.

The matrix of T with respect to the ordered basis $\mathcal{B} = \{(1,3),(2,5)\}$ is

[Question ID = 10220]

1. $\begin{bmatrix} 121 & 201 \\ -70 & -116 \end{bmatrix}$

   [Option ID = 40874]
2. $\begin{bmatrix} -121 & 201 \\ 70 & -116 \end{bmatrix}$

   [Option ID = 40875]
3. $\begin{bmatrix} -121 & 201 \\ 70 & 116 \end{bmatrix}$

   [Option ID = 40876]
4. $\begin{bmatrix} -121 & -201 \\ 70 & 116 \end{bmatrix}$

   [Option ID = 40877]

Correct Answer :-

- $\begin{bmatrix} 121 & 201 \\ -70 & -116 \end{bmatrix}$

  [Option ID = 40874]

16) Let f be a bounded Lebesgue measurable function on $[\pi, 3\pi]$. Then $\lim_{n\to\infty} \int_{\pi}^{3\pi} f(x) \sin nx \, dx$

[Question ID = 10221]

1. may not exist

   [Option ID = 40878]
2. exists and is equal to 1

   [Option ID = 40879]
3. exists and is equal to 0

   [Option ID = 40880]
4. exists and is equal to 2

   [Option ID = 40881]

Correct Answer :-

- exists and is equal to 0

  [Option ID = 40880]

17) Let $A = \left\{x \in (0,1) : \sin\left(\frac{1}{x}\right) = 0\right\}$ and $f : [0,1] \to \mathbb{R}$ be given by

$f(x) = \begin{cases} \frac{1}{\sin\left(\frac{1}{x}\right)}, & \text{if } x \in (0,1) \sim A, \end{cases}$

Which of the following statements is true?

[Question ID = 10222]

1. f is not Lebesgue measurable

   [Option ID = 40882]
2. f is Lebesgue measurable but $|f|$ is not Lebesgue measurable

   [Option ID = 40883]
3. f is Lebesgue measurable but not Lebesgue integrable on $[0,1]$

   [Option ID = 40884]
4. f is Lebesgue integrable on $[0,1]$

   [Option ID = 40885]

Correct Answer :-

- f is Lebesgue integrable on $[0,1]$

  [Option ID = 40885]

18) Let $z = x + iy \in \mathbb{C}$ and

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & \text{if } z \neq 0, \\ 0, & \text{otherwise.} \end{cases}$$

If $f(z) = u(x,y) + iv(x,y)$, then at origin

[Question ID = 10223]

1. u and v do not satisfy Cauchy Riemann equations but f is differentiable

   [Option ID = 40886]
2. u and v do not satisfy Cauchy Riemann equations and f is not differentiable

   [Option ID = 40887]
3. u and v satisfy Cauchy Riemann equations but f is not differentiable

   [Option ID = 40888]
4. u and v satisfy Cauchy Riemann equations and f is differentiable

   [Option ID = 40889]

Correct Answer :-

- u and v satisfy Cauchy Riemann equations but f is not differentiable

  [Option ID = 40888]

19) The value of the integral $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the anticlockwise circle $|z| = 3$ is

[Question ID = 10224]

1. $2\pi(e^2 + 2e^4)i$

   [Option ID = 40890]
2. $2\pi(e^4 - e^2)i$

   [Option ID = 40891]
3. $2\pi(e^2 - e^4)i$

   [Option ID = 40892]
4. $2\pi(e^2 - 2e^4)i$

   [Option ID = 40893]

Correct Answer :-

- $2\pi(e^4 - e^2)i$

  [Option ID = 40891]

20) For $n \in \mathbb{N}$, let $z_n = (-1)^n$ and $w_n = \begin{cases} \frac{1}{n}, & \text{if } n \text{ is even,} \\ 0, & \text{if } n \text{ is odd.} \end{cases}$

Let $G = \{z \in \mathbb{C} : |z| < 2\}$ and $f, g : G \to \mathbb{C}$ be functions such that $f\left(\frac{1}{n}\right) = z_n$ and $g\left(\frac{1}{n}\right) = w_n$ for all $n \in \mathbb{N}$. Then on G

[Question ID = 10225]

1. f can be chosen to be analytic but g cannot be analytic

   [Option ID = 40894]
2. g can be chosen to be analytic but f cannot be analytic

   [Option ID = 40895]
3. Neither f nor g can be analytic

   [Option ID = 40896]
4. Both f and g can be chosen to be analytic

   [Option ID = 40897]

Correct Answer :-

- Neither f nor g can be analytic

  [Option ID = 40896]

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21) Let $f(z) = a_0 + a_1 z + \cdots + a_{20} z^{20}$, $z \in \mathbb{C}$ be such that $|f(z)| \le 1$, for $|z| \le 1$. Then for all $n = 1,2,\ldots,20$,

[Question ID = 10226]

1. $|a_n| \le 1$ and f is a constant

   [Option ID = 40898]
2. $|a_n| \le 1$ but f need not be a constant

   [Option ID = 40899]
3. $1 < |a_n| < n$ but f need not be a constant

   [Option ID = 40900]
4. $1 < |a_n| < n$ and f is a constant

   [Option ID = 40901]

Correct Answer :-

- $|a_n| \le 1$ but f need not be a constant

  [Option ID = 40899]

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22) Let S be compact subset of a metric space. Then,

[Question ID = 10227]

1. S is complete and totally bounded

   [Option ID = 40902]
2. S is complete but need not be totally bounded

   [Option ID = 40903]
3. S is totally bounded but need not be complete

   [Option ID = 40904]
4. S is totally bounded but $\bar{S}$ need not be totally bounded

   [Option ID = 40905]

Correct Answer :-

- S is complete and totally bounded

  [Option ID = 40902]

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23) Which of the following statements is false?

[Question ID = 10228]

1. The connected subsets of $\mathbb{R}$, with the usual metric, are precisely its intervals

   [Option ID = 40906]
2. The real line $\mathbb{R}$ and the Euclidean plane $\mathbb{R}^2$ are not homeomorphic

[Option ID = 40907]

3. If A is a connected metric space then every continuous function $f : A \to \mathbb{N}$ must be constant.

[Option ID = 40908]

4. If $B \subseteq \mathbb{R}$ is bounded and $f : \mathbb{R} \to \mathbb{R}$ is continuous on B then $f(B)$ is bounded

[Option ID = 40909]

Correct Answer :-

- If $B \subseteq \mathbb{R}$ is bounded and $f : \mathbb{R} \to \mathbb{R}$ is continuous on B then $f(B)$ is bounded

[Option ID = 40909]

24) Let $\{X_\alpha : \alpha \in \Lambda\}$ be a family of topological spaces. Which of the following statements is false?

[Question ID = 10229]

1. Product topology on $\prod_{\alpha\in\Lambda} X_\alpha$ is finer than the box topology on $\prod_{\alpha\in\Lambda} X_\alpha$

[Option ID = 40910]

2. If $\prod_{\alpha\in\Lambda} X_\alpha$ has product topology and $E_\alpha \subseteq X_\alpha$ then $\overline{\prod_{\alpha\in\Lambda} E_\alpha} = \prod_{\alpha\in\Lambda} \overline{E_\alpha}$.

[Option ID = 40911]

3. If each X_α is completely regular then $\prod_{\alpha\in\Lambda} X_\alpha$ under product topology is completely regular

[Option ID = 40912]

4. If $\mathcal{B}_\alpha$ is a basis for X_α then the family of sets $\prod_{\alpha\in\Lambda} B_\alpha, B_\alpha \in \mathcal{B}_\alpha$ is a basis for $\prod_{\alpha\in\Lambda} X_\alpha$ under box topology

[Option ID = 40913]

Correct Answer :-

- Product topology on $\prod_{\alpha\in\Lambda} X_\alpha$ is finer than the box topology on $\prod_{\alpha\in\Lambda} X_\alpha$

[Option ID = 40910]

25) Which of the following statements is false?

[Question ID = 10230]

1. The open continuous image of a first countable space is first countable.

[Option ID = 40914]

2. The space $\mathbb{R}_l$ ($\mathbb{R}$ with lower limit topology) is separable but not second countable.

[Option ID = 40915]

3. Let X and Y be topological spaces, $f : X \to Y$ be such that $x_n \to x$ in X implies $f(x_n) \to f(x)$ in Y, then f is continuous.

[Option ID = 40916]

4. Both axioms of first countability and second countability are hereditary.

[Option ID = 40917]

Correct Answer :-

- Let X and Y be topological spaces, $f : X \to Y$ be such that $x_n \to x$ in X implies $f(x_n) \to f(x)$ in Y, then f is continuous.

[Option ID = 40916]

26) Which of the following statements is false?

[Question ID = 10231]

1. All the cubes, spheres and discs are compact in $\mathbb{R}^n$.

[Option ID = 40918]

2. For a metric space (X, d) and $x \in X$, $\overline{\{y \in X : d(x,y) < r\}} = \{y \in X : d(x,y) \le r\}$.

[Option ID = 40919]

3. If $(A_n)_{n\in\mathbb{N}}$ is a sequence of nowhere dense sets in a complete metric space X then $X \neq \bigcup_{n\in\mathbb{N}} A_n$.

[Option ID = 40920]

4. Any continuous function from $[a, b]$ to $\mathbb{R}$ is limit of some uniformly convergent sequence of polynomials.

[Option ID = 40921]

Answer :-

- a metric space (X,d) and $x \in X$, $\{y \in X : d(x,y) < r\} = \{y \in X : d(x,y) \leq r\}$.

[Option ID = 40919]

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27) Which of the following statements is false?

[Question ID = 10232]

1. The space of all real valued continuous functions on $[a,b]$ under supremum norm is separable.

   [Option ID = 40922]
2. Every subset of the Euclidean space $\mathbb{R}^n$ is separable.

   [Option ID = 40923]
3. Every compact metric space is separable.

   [Option ID = 40924]
4. A closed subspace of a separable topological space is separable.

   [Option ID = 40925]

Correct Answer :-

- A closed subspace of a separable topological space is separable.

[Option ID = 40925]

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28) Let $(X, \|.\|_X)$ and $(Y, \|.\|_Y)$ be two normed spaces. For $(x,y) \in X \times Y$ define $\|(x,y)\|_1 = \|x\|_X + \|y\|_Y$; $\|(x,y)\|_2 = \left(\|x\|_X{}^{1/2} + \|y\|_Y{}^{1/2}\right)^2$; $\|(x,y)\|_3 = \left(\|x\|_X{}^3 + \|y\|_Y{}^3\right)^{1/3}$.

Consider the following statements:

a. $\|.\|_1$ defines a norm on $X \times Y$.
b. $\|.\|_2$ defines a norm on $X \times Y$.
c. $\|.\|_3$ defines a norm on $X \times Y$.

Which of the following options is correct?

[Question ID = 10233]

1. Only a) and b) are correct.

   [Option ID = 40926]
2. Only a) and c) are correct.

   [Option ID = 40927]
3. Only b) and c) are correct.

   [Option ID = 40928]
4. None of a), b) and c) is correct.

   [Option ID = 40929]

Correct Answer :-

- Only a) and c) are correct.

[Option ID = 40927]

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29) For a normed space X consider the following statements:

a. For a sequence $(x_n)_{n\in\mathbb{N}}$ in X if $\sum_{n=1}^{\infty}\|x_n\| < \infty$, then the series $\sum_{n=1}^{\infty} x_n$ converges in X.
b. If X is complete and for a sequence $(x_n)_{n\in\mathbb{N}}$ in X if $\sum_{n=1}^{\infty}\|x_n\| < \infty$, then the series $\sum_{n=1}^{\infty} x_n$ converges in X.
c. If $\sum_{n=1}^{\infty}\|x_n\| < \infty$ implies the series $\sum_{n=1}^{\infty} x_n$ converges for any sequence $(x_n)_{n\in\mathbb{N}}$ in X, then X is complete.
d. For a sequence $(x_n)_{n\in\mathbb{N}}$ in X if the series $\sum_{n=1}^{\infty} x_n$ converges in X, then $\sum_{n=1}^{\infty}\|x_n\| < \infty$.

Which of the following options is correct?

[Question ID = 10234]

1. Only a) and b) are correct.

   [Option ID = 40930]

3. Only a) and c) are correct.

[Option ID = 40932]

4. Only b) and d) are correct.

[Option ID = 40933]

Correct Answer :-

- Only b) and c) are correct.

[Option ID = 40931]

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30) For a normed space X let X^{**} denote the second dual of X. Consider the following statements:

a. $C_0^{**} \approx l_\infty$.
b. $l_2^{**} \approx l_2$.
c. $l_1^{**} \approx C_0$.
d. $l_4^{**} \approx l_{4/3}$.

(C_0 being the space of all sequences converging to 0 and l_p $(p \geq 1)$ the sequence space of p-summable sequences).

Which of the following options is correct?

[Question ID = 10235]

1. Only a) and c) are correct.

[Option ID = 40934]

2. Only a) and b) are correct.

[Option ID = 40935]

3. Only b) and d) are correct.

[Option ID = 40936]

4. Only c) and d) are correct.

[Option ID = 40937]

Correct Answer :-

- Only a) and b) are correct.

[Option ID = 40935]

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31) Let $H = L^2[0,2\pi]$ and $\mathcal{B} = \left\{\frac{1}{\sqrt{2\pi}} e_n : n \in \mathbb{Z}\right\}$, where $e_n(t) = e^{int}$, $t \in [0,2\pi]$.

Consider the following statements:

a. $\mathcal{B}$ is a Hamel basis for H.
b. $\mathcal{B}$ is an orthonormal set in H.
c. $\mathcal{B}$ is a complete orthonormal set in H.
d. H is not separable.

Which of the following options is correct?

[Question ID = 10236]

1. Only a) is correct.

[Option ID = 40938]

2. Only b) and d) are correct.

[Option ID = 40939]

3. Only b) and c) are correct.

[Option ID = 40940]

4. Only c) and d) are correct.

[Option ID = 40941]

Correct Answer :-

- Only b) and c) are correct.

[Option ID = 40940]

Let G be a finite group and $A, B \subseteq G$ be such that $|A| + |B| > |G|$ (where $|A|$ denotes the number of elements of A). Then $A.B = \{a.b : a \in A, b \in B\}$ is

[Question ID = 10237]

1. a proper subgroup of G.

   [Option ID = 40942]
2. not a subgroup of A.

   [Option ID = 40943]
3. equal to G.

   [Option ID = 40944]
4. $|A.B| < |G|$.

   [Option ID = 40945]

Correct Answer :-

- equal to G.

  [Option ID = 40944]

33) Let f, g and h be polynomials over $\mathbb{Q}$ given by $f(x) = x^n + n$, where n is a positive integer, $g(x) = x^5 - 5x - 2$ and $h(x) = x^4 - 2x^2 + 1$. Which of the following statements is true?

[Question ID = 10238]

1. Only $f(x)$ and $g(x)$ are irreducible over $\mathbb{Q}$.

   [Option ID = 40946]
2. Only $g(x)$ and $h(x)$ are irreducible over $\mathbb{Q}$.

   [Option ID = 40947]
3. Only $f(x)$ and $h(x)$ are irreducible over $\mathbb{Q}$.

   [Option ID = 40948]
4. All of $f(x), g(x)$ and $h(x)$ are irreducible over $\mathbb{Q}$.

   [Option ID = 40949]

Correct Answer :-

- Only $f(x)$ and $g(x)$ are irreducible over $\mathbb{Q}$.

  [Option ID = 40946]

34) For any pair of real numbers $a(a \neq 0)$ and b, define a function $f_{a,b} : \mathbb{R} \to \mathbb{R}$ by $f_{a,b}(x) = ax + b$. Consider the following statements:

a. The function $f_{a,b}$ is a permutation of $\mathbb{R}$.

b. $f_{a,b} \circ f_{c,d} = f_{ac,ad+b}$

c. $G = \{f_{a,b} : a \in \mathbb{R} \sim \{0\}, b \in \mathbb{R}\}$ forms a group under composition.

Which of the following options is correct?

[Question ID = 10239]

1. Only a) and b) are correct.

   [Option ID = 40950]
2. Only a) and c) are correct.

   [Option ID = 40951]
3. All of a), b) and c) are correct.

   [Option ID = 40952]
4. Only b) and c) are correct.

   [Option ID = 40953]

Correct Answer :-

- All of a), b) and c) are correct.

  [Option ID = 40952]

Which of the following rings has a finite group of units?

[Question ID = 10240]

1. $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$

[Option ID = 40954]

2. $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}\}$

[Option ID = 40955]

3. $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$

[Option ID = 40956]

4. $\mathbb{Q}[i] = \{a + bi : a, b \in \mathbb{Q}\}$

[Option ID = 40957]

Correct Answer :-

- $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}\}$

[Option ID = 40955]

36) How many irreducible quadratics (degree 2) are there over a finite field having n elements?

[Question ID = 10241]

1. $n^2(n+1)$

[Option ID = 40958]

2. $n(n-1)^2$

[Option ID = 40959]

3. $\frac{n^2(n-1)}{2}$

[Option ID = 40960]

4. $\frac{n(n-1)^2}{2}$

[Option ID = 40961]

Correct Answer :-

- $\frac{n^2(n-1)}{2}$

[Option ID = 40960]

37) Which of the following vector spaces has dimension not divisible by 2?

[Question ID = 10242]

1. A plane passing through origin in $\mathbb{R}^3$ over $\mathbb{R}$.

[Option ID = 40962]

2. The set P_3 of all polynomials over $\mathbb{R}$ of degree ≤ 3

[Option ID = 40963]

3. $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ over $\mathbb{Z}_3$.

[Option ID = 40964]

4. $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ over $\mathbb{Z}_2$.

[Option ID = 40965]

Correct Answer :-

- $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ over $\mathbb{Z}_2$.

[Option ID = 40965]

38) Let L be the line passing through the origin and $(1,1)$ in $\mathbb{R}^2$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x,y)$ = projection of (x,y) on L.

Then the eigenvalues of T are

...tion ID = 10243]

1. ...d $\frac{1}{2}$.

[Option ID = 40966]

2. 0 and 1.

[Option ID = 40967]

3. 0 and $\frac{1}{2}$.

[Option ID = 40968]

4. 1 and $\frac{1}{4}$.

[Option ID = 40969]

Correct Answer :-

- 0 and 1.

[Option ID = 40967]

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39) A particular integral of the partial differential equation

$$\left(\frac{\partial}{\partial x}-3\frac{\partial}{\partial y}-2\right)^2 z = e^{2x}\sin(y+3x)$$

is

[Question ID = 10244]

1. $\frac{1}{2}x^2e^{2x}\sin(y+3x)$

[Option ID = 40970]

2. $\frac{1}{2}xe^{2x}\cos(y+3x)$

[Option ID = 40971]

3. $\frac{1}{2}x^3e^{2x}\sin(y+3x)$

[Option ID = 40972]

4. $\frac{1}{2}xe^{2x}[\sin(y+3x)+\cos(y+3x)]$

[Option ID = 40973]

Correct Answer :-

- $\frac{1}{2}x^2e^{2x}\sin(y+3x)$

[Option ID = 40970]

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40) The partial differential equation

$$x(y-z)\frac{\partial z}{\partial x}+y(z-x)\frac{\partial z}{\partial y}=z(x-y)$$

has general solution (with arbitrary function φ)

[Question ID = 10245]

1. $\varphi(x+y+z, xyz)=0.$

[Option ID = 40974]

2. $\varphi\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}, xyz\right)=0$

[Option ID = 40975]

3. $\varphi(x^2+y^2, xyz)=0.$

[Option ID = 40976]

4. $\varphi(x^3+y^3+x+y, xyz)=0.$

[Option ID = 40977]

Correct Answer :-

- $\varphi(x+y+z, xyz)=0.$

ion ID = 40974]

The general solution of the differential equation

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin(\log(1+x))$$

is (with arbitrary constants c_1 and c_2)

[Question ID = 10246]

1. $c_1\cos(\log(1+x))+c_2\sin(\log(1+x))+\log(1+x)\sin(\log(1+x))$

   [Option ID = 40978]
2. $c_1\cos(\log(1+x))+c_2\sin(\log(1+x))-\log(1+x)\cos(\log(1+x))$

   [Option ID = 40979]
3. $c_1\cos(\log(1+x))+c_2\sin(\log(1+x))+\log(1+x)$

   [Option ID = 40980]
4. $(c_1+c_2\log(1+x))\cos(\log(1+x))-\log(1+x)\sin(\log(1+x))$

   [Option ID = 40981]

Correct Answer :-

- $c_1\cos(\log(1+x))+c_2\sin(\log(1+x))-\log(1+x)\cos(\log(1+x))$

  [Option ID = 40979]

42) For the initial value problem

$$\frac{dy}{dx}=f(x,y),\qquad y(0)=0,$$

which of the following statements is true?

[Question ID = 10247]

1. $f(x,y)=\sqrt{y}$ satisfies Lipschitz condition and the above problem has a unique solution.

   [Option ID = 40982]
2. $f(x,y)=y^{2/3}$ satisfies Lipschitz condition and the above problem has a unique solution.

   [Option ID = 40983]
3. $f(x,y)=x^2|y|$, the above problem has a unique solution.

   [Option ID = 40984]
4. $f(x,y)=e^y$, the above problem has at least two solutions.

   [Option ID = 40985]

Correct Answer :-

- $f(x,y)=x^2|y|$, the above problem has a unique solution.

  [Option ID = 40984]

43) The solution of the initial boundary value problem

$$u_{tt}-c^2u_{xx}=0,\qquad 0<x<L,\qquad t>0,$$

$$u_x(0,t)=x,\qquad u_x(L,t)=0,$$

$$u(x,0)=x,\qquad u_t(x,0)=0,$$

is

[Question ID = 10248]

1. $u(x,t)=\frac{L}{2}+\sum_{n=1}^{\infty}\left[\frac{2}{L}\left(\frac{L}{n\pi}\right)^2((-1)^n-1)\cos\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}\right]$

   [Option ID = 40986]
2. $u(x,t)=L+\sum_{n=1}^{\infty}\left[L\left(\frac{L}{n\pi}\right)^2((-1)^n-1)\cos\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}\right]$

   [Option ID = 40987]

3. $u(x,t)$ [illegible] $\sum_{n=1}^{\infty}\left[\frac{L}{[illegible]}\left(\frac{L}{n\pi}\right)^2((-1)^n-1)\cos\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}\right].$

[Option ID = 40988]

4. $u(x,t) = \frac{2}{L}+\sum_{n=1}^{\infty}\left[\frac{L}{2}\left(\frac{n\pi}{L}\right)^2((-1)^n-1)\cos\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}\right].$

[Option ID = 40989]

Correct Answer :-

- $u(x,t) = \frac{L}{2}+\sum_{n=1}^{\infty}\left[\frac{2}{L}\left(\frac{L}{n\pi}\right)^2((-1)^n-1)\cos\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}\right]$

[Option ID = 40986]

44) The solution of the differential equation $uu_t + u_x = -u$,
$u(0,t) = \alpha t$,

where α is a constant, is

[Question ID = 10249]

1. $u(x,t) = \frac{t\alpha e^{x}}{1+\alpha-\alpha e^{-x}}.$

[Option ID = 40990]

2. $u(x,t) = \frac{t\alpha e^{-x}}{1-\alpha+\alpha e^{-x}}.$

[Option ID = 40991]

3. $u(x,t) = \frac{t\alpha e^{-x}}{1+\alpha-\alpha e^{-x}}.$

[Option ID = 40992]

4. $u(x,t) = \frac{t\alpha e^{x}}{1-\alpha+\alpha e^{-x}}.$

[Option ID = 40993]

Correct Answer :-

- $u(x,t) = \frac{t\alpha e^{-x}}{1+\alpha-\alpha e^{-x}}.$

[Option ID = 40992]

45) The eigen values λ_n and the eigen functions φ_n of the Sturm-Liouville problem

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\lambda y = 0, \quad 1 \le x \le e,$$

$$y(1) = 0, \quad y(e) = 0,$$

are given by

[Question ID = 10250]

1. $\lambda_n = n^2, \varphi_n(x) = \sin(\log x), \quad n = 1,2,3 \dots .$

[Option ID = 40994]

2. $\lambda_n = n^2\pi^2, \varphi_n(x) = \cos(n\pi \log x), \quad n = 1,2,3 \dots .$

[Option ID = 40995]

3. $\lambda_n = n^2\pi^2, \varphi_n(x) = \sin(n\pi \log x), \quad n = 1,2,3 \dots .$

[Option ID = 40996]

4. $\lambda_n = n^2, \varphi_n(x) = \cos(\log x), \quad n = 1,2,3 \dots .$

[Option ID = 40997]

Correct Answer :-

- $\lambda_n = n^2\pi^2, \varphi_n(x) = \sin(n\pi \log x), \quad n = 1,2,3 \dots .$

[Option ID = 40996]

46) The general solution of the Laplace equation

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta} = 0, \quad 0 \le r < a, \quad 0 < \theta \le 2\pi$$

is [illegible]

[Question ID = 10251]

1. $u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^n (a_n \cos n\theta + b_n \sin n\theta).$

   [Option ID = 40998]
2. $u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^n (a_n \cos n\theta + b_n \sin n\theta).$

   [Option ID = 40999]
3. $u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^n (a_n e^{n\theta} + b_n e^{-n\theta}).$

   [Option ID = 41000]
4. $u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^n (a_n e^{n\theta} + b_n e^{-n\theta}).$

   [Option ID = 41001]

Correct Answer :-

- $u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^n (a_n \cos n\theta + b_n \sin n\theta).$

  [Option ID = 40998]

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47) Consider the motion of an incompressible inviscid fluid moving under an arbitrary body force $\vec{F}$ per unit mass with velocity $\vec{q}$. The generation of vorticity $\vec{w}$ is given by

[Question ID = 10252]

1. $\frac{d\vec{w}}{dt} = (\vec{w}.\vec{\nabla})\vec{w} + \text{curl}\,\vec{F}.$

   [Option ID = 41002]
2. $\frac{d\vec{w}}{dt} = (\vec{w}.\vec{\nabla})\vec{F} + \vec{w}.$

   [Option ID = 41003]
3. $\frac{d\vec{w}}{dt} = (\vec{w}.\vec{\nabla})\vec{q} + \text{curl}\,\vec{F}.$

   [Option ID = 41004]
4. $\frac{d\vec{w}}{dt} = \vec{q} + \vec{F}.$

   [Option ID = 41005]

Correct Answer :-

- $\frac{d\vec{w}}{dt} = (\vec{w}.\vec{\nabla})\vec{q} + \text{curl}\,\vec{F}.$

  [Option ID = 41004]

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48) The Navier-Stokes equation for steady, viscous incompressible flow under no body force with $\vec{q}$ as velocity, $\vec{w}$ as vorticity vector, p as pressure, ρ as density, ν as viscosity, may be developed in the form

[Question ID = 10253]

1. $\vec{q} \times \vec{w} = \nabla\left(\frac{1}{2}q^2 + \frac{p}{\rho}\right) + \nu\,\text{curl}\,\vec{w}.$

   [Option ID = 41006]
2. $\vec{q} \times \vec{w} = \nu\,\text{curl}\,\vec{q}.$

   [Option ID = 41007]
3. $\vec{q} \times \vec{w} = \nabla\left(\frac{1}{2}q^2 + \frac{p}{\rho}\right) + \text{curl}\,\vec{w}.$

   [Option ID = 41008]
4. $\vec{q} \times \vec{w} = \nabla\left(\frac{1}{2}w^2 + \frac{p}{\rho}\right) + \text{curl}\,\vec{q}.$

   [Option ID = 41009]

Correct Answer :-

- $\vec{q} \times \vec{w} = \nabla\left(\frac{1}{2}q^2 + \frac{p}{\rho}\right) + \nu\,\text{curl}\,\vec{w}.$

on ID = 41006]

49) Consider a solid stationary sphere of radius a placed in a uniform stream of liquid for which the undisturbed velocity is $-U\hat{i}$, where U is a constant. Then the velocity component for $r \geq a$ is given by

[Question ID = 10254]

1. $q_\theta = U \sin\theta \left(1 + \frac{a^3}{2r^3}\right)$.

   [Option ID = 41010]
2. $q_\theta = U \sin\theta \left(1 - \frac{a^3}{2r^2}\right)$.

   [Option ID = 41011]
3. $q_x = -U \sin\theta$.

   [Option ID = 41012]
4. $q_r = 0$.

   [Option ID = 41013]

Correct Answer :-

- $q_\theta = U \sin\theta \left(1 + \frac{a^3}{2r^3}\right)$.

  [Option ID = 41010]

50) Let R be a commutative ring with unity and $f(x) = \sum_{i=0}^{n} a_i x^i \in R[x]$. Then $f(x)$ is a unit in $R[x]$ if and only if

[Question ID = 10255]

1. a_0 is a unit and a_i $(1 \leq i \leq n)$ are nilpotents in R .
   [Option ID = 41014]
2. a_i $(0 \leq i \leq n)$ are units in R .

   [Option ID = 41015]
3. a_i $(0 \leq i \leq n)$ are nilpotents in R .
   [Option ID = 41016]
4. a_i $(0 \leq i \leq n)$ are zero divisors in R .

   [Option ID = 41017]

Correct Answer :-

- a_0 is a unit and a_i $(1 \leq i \leq n)$ are nilpotents in R .
  [Option ID = 41014]