



USN

15MAT31

**Third Semester B.F. Degree Examination, Dec..24\*Jan.2020**  
**Engineering Mathematics — III**

Time: 3 hrs.

Max. Marks: 80

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

Module-I

- 1 a. Obtain the Fourier expansion of the function  $f(x) = x$  over the interval  $(-7c, 7c)$ . Deduce that  $\frac{1}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$  (08 Marks)
- b. The following table gives the variations of a periodic current A over a certain period T:

| t (sec) | 0    | T/6  | T/3  | T/2  | 2T/3  | 5T/6  | T    |
|---------|------|------|------|------|-------|-------|------|
| A (amp) | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the first harmonic. (08 Marks)

**OR**

- 2 a. Obtain the Fourier series for the function  $f(x) = \begin{cases} x, & \text{for } 0 < x < \pi/2 \\ \pi/2, & \text{for } \pi/2 < x < \pi \end{cases}$  (06 Marks)
- b. Represent the function  $f(x) = \begin{cases} x, & \text{for } 0 < x < \pi/2 \\ \pi/2, & \text{for } \pi/2 < x < \pi \end{cases}$  in a half range Fourier sine series. (05 Marks)
- c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following table:

| $x^\circ$ | 0 | 45  | 90 | 135 | 180 | 225 | 270 | 315 |
|-----------|---|-----|----|-----|-----|-----|-----|-----|
| y         | 2 | 3/2 | 1  | 1/2 | 0   | 1/2 | 1   | 3/2 |

(05 Marks)

Module-2

- 3 Find the complex Fourier transform of the function  $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$ . Hence evaluate  $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$ . (06 Marks)
- b. If  $u(z) = \frac{3z+12}{(z-1)^2}$  show that  $u_0 = 0$ ,  $u_1 = 0$ ,  $u_2 = 2$ ,  $u_3 = 11$ . (05 Marks)
- c. Obtain the Fourier cosine transform of the function  $f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$ . (05 Marks)

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OR

- 4 a. Obtain the Z-transform of  $\cos n\theta$  and  $\sin n\theta$ . (06 Marks)
- b. Find the Fourier sine transform of  $f(x) = \frac{x}{1+x^2}$  and hence evaluate  $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$   $m > 0$ . (05 Marks)
- c. Solve by using Z-transforms  $y_{n+2} + 2y_{n+1} + y_n = n$  with  $y_0 = 0 = y_1$ . (05 Marks)

**Module-3**

- 5 a. Fit a second degree parabola  $y = ax^2 + bx + c$  in the least square sense for the following data and hence estimate  $y$  at  $x = 6$ . (06 Marks)

|   |    |    |    |    |    |
|---|----|----|----|----|----|
|   |    | 2  | 3  | 4  | 5  |
| y | 10 | 12 | 13 | 16 | 19 |

- b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

|   |   |   |    |   |    |    |    |    |    |    |
|---|---|---|----|---|----|----|----|----|----|----|
| x | 1 | 3 | 4  | 2 | 5  | 8  | 9  | 10 | 13 | 15 |
| y | 8 | 6 | 10 | 8 | 12 | 16 | 16 | 10 | 32 | 32 |

- c. Use Newton-Raphson method to find a real root of  $x \sin x + \cos x = 0$  near  $x = \pi$ . Carry out the upto four decimal places of accuracy. (05 Marks)

OR

- 6 a. Show that a real root of the equation  $\tan x + \tanh x = 0$  lies between 2 and 3. Then apply the Regula Falsi method to find third approximation. (06 Marks)
- b. Compute the coefficient of correlation and the equation of the lines of regression for the data:

|   |   |   |    |    |    |    |    |
|---|---|---|----|----|----|----|----|
| x | 1 | 2 | 3  | 4  | 5  | 6  | 7  |
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

- c. Fit a curve of the form  $y = ae^{bx}$  for the data:

|   |      |    |       |
|---|------|----|-------|
| x | 0    | 2  | 4     |
| y | 8.12 | 10 | 31.82 |

**Module-4**

- 7 a. From the following table find the number of students who have obtained:
- Less than 45 marks
  - Between 40 and 45 marks.

|                    |       |       |       |       |       |
|--------------------|-------|-------|-------|-------|-------|
| Marks              | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Number of students | 31    | 42    | 51    | 35    | 31    |

- b. Construct the interpolating polynomial for the data given below using Newton's general interpolation formula for divided differences and hence find  $y$  at  $x = 3$ . (06 Marks)

|   |    |    |     |     |     |      |
|---|----|----|-----|-----|-----|------|
| x | 2  | 4  | 5   | 6   | 8   | 10   |
| y | 10 | 96 | 196 | 350 | 868 | 1746 |

- c. Evaluate  $\int_0^1 \frac{x}{1+x^2} dx$  by Weddle's rule. Taking seven ordinates. Hence find  $\log_e 2$ . (05 Marks)

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**OR**

- 8 a. Use Lagrange's interpolation formula to find  $f(4)$  given below. (06 Marks)

|        |   |    |    |     |
|--------|---|----|----|-----|
|        |   |    |    | 6   |
| $f(x)$ | 1 | 11 | 14 | 158 |

- b. Use Simpson's  $3/8$  rule to evaluate  $\int_1^4 e^x dx$  (05 Marks)

- c. The area of a circle (A) corresponding to diameter (D) is given by

|   |      |      |      |      |      |
|---|------|------|------|------|------|
| D | 80   | 85   | 90   | 95   | 100  |
| A | 5026 | 5674 | 6362 | 7088 | 7854 |

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

(05 Marks)

### Module-5

- 9 a. Evaluate Green's theorem for  $\int_C (xy + y^2) dx + x^2 dy$  where  $C$  is the closed curve of the region bounded by  $y = x$  and  $y = x^2$ . (06 Marks)
- b. Find the extrema of the functional  $f(x, y, z) = x^2 + 3y^2 + 2z^2 + 2xy$ . (05 Marks)
- c. Verify Stoke's theorem for  $\mathbf{F} = (2x - y)\mathbf{i} + yz^2\mathbf{j} - y^2z\mathbf{k}$  where  $S$  is the upper half surface of the sphere  $x^2 + y^2 + z^2 = 1$ ,  $C$  is its boundary. (05 Marks)

**OR**

- 10 a. Derive Euler's equation in the standard form  $\frac{d}{dx} \left( \frac{\partial f}{\partial y_1} \right) = \frac{\partial f}{\partial y_1}$ . (06 Marks)
- b. If  $\mathbf{F} = 2xy\mathbf{i} + 3x^2z\mathbf{j} + xz\mathbf{k}$  and  $S$  is the rectangular parallelepiped bounded by  $x = 0, y = 0, z = 0, x = 2, y = 1, z = 3$ . Evaluate  $\oint_C \mathbf{F} \cdot d\mathbf{r}$ . (05 Marks)
- c. Prove that the shortest distance between two points in a plane is along the straight line joining them or prove that the geodesics on a plane are straight lines. (05 Marks)