Code: 17F00101

MCA I Semester Supplementary Examinations June/July 2018

MATHEMATICAL FOUNDATIONS FOR COMPUTER SCIENCE

(For students admitted in 2017 only)

Time: 3 hours Max. Marks: 60

Answer all the questions

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1 (a) Write a note on mathematical induction.

(b) Show that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$ by mathematical induction.

(c) Show that any positive integer n greater-than or equal to 2 is either a prime or a product of prime.

OR

2 (a) Let R be a binary relation on the set of all positive integers such that;
$R = \{(a,b)/a - b \text{ is an odd positive integer}\}$. Show that R is an equivalence relation.

(b) Prove that $P \vee Q \Leftrightarrow \neg\ (\neg P \wedge \neg Q)$ using truth table.

3 (a) State and prove Lagrange's theorem.

(b) Let $(A,\ *)$ be an algebraic system where $*$ is a binary operation such that for any a & b in A, a$*$b = a.
(i) Show that $*$ is a associative operation.
(ii) Can $*$ ever be a commutative operation.

OR

4 (a) Show that any subgroup of a cyclic group is cyclic.

(b) Show that every group containing exactly two elements is isomorphic to $(Z_2,\ \oplus)$.

5 (a) Write down the rules of sum and product.

(b) In how many ways can two integers be selected from the integers 1, 2,…..100 so that their difference is exactly seven.

(c) In how many ways can two adjacent squares be selected from 8 x 8 chessboard.

(d) Five boys and five girls are to be seated in a row. In how many ways can they be seated if:
(i) All boys must be seated in the five left most seats.
(ii) No boys can be seated together.

OR

6 (a) Solve the recurrence relation:
$a_r - 7a_{r-1} + 10a_{r-2} = 0$ given that $a_0 = 0\ \&\ a_1 = 3.$

(b) Determine the particular solution for the difference equation: $a_r - 3a_{r-1} + 2a_{r-2} = 2^r$.

(c) Write a note on total solutions.

7 (a) In a directed or undirected graph with a vertices, if there is a path from vertex V_1 to vertex V_2 then there is a path of no more than n-1 edges from vertex V_1 to vertex V_2. Prove this theorem.

(b) Briefly discuss about shortest paths in weighted graph.

OR

8 (a) There is always a Hamiltonian path in a directed complete graph. Prove this theorem.

(b) Briefly discuss about operations on graphs.

9 (a) State and explain the Kruskal's algorithm with an example.

(b) Briefly discuss about binary search tree.

OR

10 (a) State and explain the Prim's algorithm with an example.

(b) Let $a = 3^r$
$b = 2^r$
(i) Does a dominate b asymptotically.
(ii) Does b dominate a asymptotically.

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