

Printed Pages: 5

1327

AS-101

(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID : 199111

Roll No. 

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B.Tach.

(SEM. I) THEORY EXAMINATION, 2015-16

ENGINEERING MATHEMATICS-I

[Time : 3 hours]

[Total Marks : 100]

**SECTION-A**

1. Attempt all parts. All parts carry equal marks. Write answer of each part in shots. (10×2=20)

(a) If  $u = \log(x^2/y)$  then value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = ?$

(b) If  $z = xyf(x/y)$  then value of  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2z$

(c) Apply Taylor's series find expansion of  $f(x, y) = x^3 + xy^2$  about point(2, 1) upto first degree term.

(d) If  $x = u - v, y = u^2 - v^2$ , find the value of  $\frac{\partial(u, v)}{\partial(x, y)}$ .

(e) Find all the asymptotes of the curve  $xy^2 = 4a^2(2a - x)$

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(1)

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(f) Find the inverse of the matrix by using elementary row operation.  $A = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$

(g) If  $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$ , find the eigen value of  $A^2$

(h) Evaluate  $\int_0^1 \int_1^2 \int_2^3 xyz \, dx \, dy \, dz$ .

(i) If  $\phi(x, y, z) = x^2y + y^2x + z^2$ , find  $\nabla \phi$  at the point  $(1, 1, 1)$ .

(j) Evaluate  $\frac{r(8/3)}{r(2/3)}$ .

### SECTION-B

Attempt any five from this section.

(10x5=50)

2. If  $x = \sin\left\{\frac{1}{m}\sin^{-1}y\right\}$  find the value of  $y_n$  at  $x=0$

3. if  $u, v, w$  are the roots of the equation  $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$  in  $\lambda$  find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ .

4000 (2)

AS-101

4. If  $r$  the distance of a point on conic  $ax^2 + by^2 + cz^2 = 1$ ,  $lx + my + nz = 0$  from origin, then that the stationary values of  $r$  are given by the equation  $\frac{l^2}{1 - ar^2} + \frac{m^2}{1 - br^2} + \frac{n^2}{1 - cr^2} = 0$

5. Find the Eigen values and corresponding Eigen

$$\text{vectors } A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

6. The plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  meets the axes in A, B, and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. Also find its mass if the density at any point is  $kxyz$ .

7. Change the order of integration in  $I = \int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$  and hence evaluate the same.

8. Verify Gauss's divergence theorem for the function  $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ , taken over the cube bounded by  $x=0$ ,  $x=1$ ,  $y=0$ ,  $y=1$  and  $z=0$ ,  $z=1$ .

9. Show that the vector field  $\vec{F} = \frac{\vec{r}}{|\vec{r}|^3}$  is irrotational as well as solenoidal. Find the scalar potential.

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**SECTION-C**

Attempt any **two** question from this section. (15x2=30)

10. (a) Expand  $e^{ax} \cos by$  in power of in powers of  $x$  and  $y$  as terms of third degree.
- (b) Determine the constant  $a$  and  $b$  such that the curl of vector.
- $$\vec{A} = (2xy + 3yz)\vec{i} + (x^2 + axz - 4z^2)\vec{j} - (3xy + byz)\vec{k}$$
- is zero.
- (c) Examine the following vector for linearly dependent and find the relation between them. If Possible.
- $$X_1 = (1, 1, -1), X_2 = (1, 1, 2, -1), X_3 = (3, 1, 0, 1).$$
11. (a) Define Beta and Gamma function and Evaluate
- $$\int_0^1 \frac{dx}{\sqrt{1+x^4}}.$$
- (b) Find the area between the parabola  $y^2 = 4ax$  and  $x^2 = 4ay$ .
- (c) If  $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$  find  $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$ .
12. (a) Evaluate  $\int_0^1 \frac{dx}{(a^x - x^a)^{1/2}}$ .

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AS-101

- (b) Reduce the matrix in to normal form and hence

find its rank

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$$

- (c) If  $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ , show that :

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0.$$

—X—

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AS-101