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(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID :199111 Roll No.

B.Tach.

(SEM. I) THEORY EXAMINATION, 2015-16

ENGINEERING MATHEMATICS-I

[Time : 3 hours] **[Tolal Marks : 100]**

SECTION-A

1. Attempt **all** parts. All parts carry equal marks. Write answer of each part in shots. (10×2=20)

   (a) If $u=\log(x^2/y)$ then value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=?$

   (b) If $z=xyf(x/y)$ then value of $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2z$

   (c) Apply Taylor's series find expansion of $f(x, y) = x^3 + xy^2$ about point(2, 1) upto first degree term.

   (d) It $x = u - v$, $y = u^2 - v^2$, find the value of $\frac{\partial(u,v)}{\partial(x,y)}$.

   (e) Find all the asymptotes of the curve $xy^2=4a^2(2a-x)$

(f) Find the inverse of the matrix by using elementary row operation. $A=\begin{bmatrix}1 & 2\\ 5 & 7\end{bmatrix}$

(g) If $A=\begin{bmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & -2\end{bmatrix}$, find the eigen value of A^2

(h) Evaluate $\int_0^1\int_1^2\int_2^3 xyz\,dx\,dy\,dz$.

(i) If $\phi(x,y,z)=x^2y+y^2x+z^2$, find $\nabla\phi$ at the point (1, 1, 1).

(j) Evaluate $\dfrac{\Gamma(8/3)}{\Gamma(2/3)}$.

SECTION-B

Attempt **any five** from this section. (10x5=50)

2. If $x=\sin\left\{\dfrac{1}{m}\sin^{-1}y\right\}$ find the value of y_n at x=0

3. if u, v, w are the roots of the equation $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in λ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$.

4. If r the distance of a point on conic $ax^2+by^2+cz^2=1$, $lx+my+nz=0$ from origin, then that the stationary values of r are given by the equation $\dfrac{l^2}{1-ar^2}+\dfrac{m^2}{1-br^2}+\dfrac{n^2}{1-cr^2}=0$

5. Find the Eigen values and corresponding Eigen vectors $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

6. The plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ meets the axes in A, B, and C. Apply Dirichlet's integral to find the volume of the tetraheadron OABC. Also find its mass if the density at any point is kxyz.

7. Change the order of Integration in $I=\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same.

8. Verify gauss's divergence theorem for the function $\vec{F}=x^2\hat{i}+z\hat{j}+yz\hat{k}$, taken over the cube bounded by x=0, x=1, y=0, y=1 and z=0, z=1.

9. Show that the vector field $\vec{F}=\dfrac{\vec{r}}{|\vec{r}|^3}$ is irrotational as well as solenoidal. Find the scalar potential.

SECTION-C

Attempt any **two** question from this section. (15x2=30)

10. (a) Expand $e^{ax}\cos by$ in power of in powers of x and y as terms of third degree.

(b) Determine the constant a and b such that the curl of vector.

$$\vec{A} = (2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} - (3xy + byz)\hat{k}$$

is zero.

(c) Examine the following vector for linearly dependent and find the relation between them. If Possible.

$X_1 = (1, 1, -1, 1), X_2 = (1, 1, 2, -1), X_3 = (3, 1, 0, 1)$.

11. (a) Define Beta and Gamma function and Evaluate

$$\int_0^1 \frac{dx}{\sqrt{1+x^4}}.$$

(b) Find the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

(c) If $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$ find $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$.

12. (a) Evaluate $\int_0^1 \frac{dx}{(a^n - x^n)^{1/n}}$.

(b) Reduce the matrix in to normal form and hence find its rank $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$

(c) It $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that :

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0.$$

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