Printed pages: 02 **Roll No.** | | | | | | | | | | | **Sub Code: RAS-103**
Paper Id: 9003

B. TECH
(SEM I) THEORY EXAMINATION 2017-18
ENGINEERING MATHE MATICS

Time: 3 Hours ***Total Marks: 70***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 7 = 14**

a. Find the n^{th} derivative of X^{n-1}log x.

b. Evaluate $\int_0^1 \int_0^{x^2} xe^y\,dxdy.$

c. If $x^2 = au + bvy^2 = au - bv$, then find $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v$.

d. Evaluate the area enclosed between the parabola y = x^2 and the straight line y = x.

e. What error in the logarithm of a number will be produced by an error of 1% in the number?`

f. Find the value of m if $\hat{F} = mxi - 5y\hat{j} + 2z\hat{k}$ is a solenoidal vector.

g. Reduce the matrix $\begin{bmatrix}1 & 1 & 1\\ 3 & 1 & 1\end{bmatrix}$ in to normal form and find its rank.

SECTION B

2. Attempt any *three parts* of the following: **7 x 3 = 21**

a) i) If u = sin $nx + cosnx, then\ prove\ that\ \ u_r = \{\,n^r 1 + (-1)^r\, sin2nx\}^{1/2}$,where u_r is the r^{th} differential coefficient of $u\ w.r.t.x.$

ii) $If\ u = sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, prove that $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y} + z\,\frac{\partial u}{\partial z} = 2\,tan\,u.$

b) i) Using elementary transformations, find the rank of the following matrix:

$$A = \begin{bmatrix}2 & -1 & 3 & -1\\ 1 & 2 & -3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}.$$

ii) Compute the inverse of the matrix $\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$ by employing elementary row transformation.

c) i) If $u_1 = \frac{x_2x_3}{x_1}, u_2 = \frac{x_3x_1}{x_2}$ and $u_3 = \frac{x_1x_2}{x_3}$ find the value of $\frac{\partial(u_1,u_2,u_3)}{\partial(x_1,x_2,x_3)}$.

ii) $If\ u = f\,(r,s,t\,), where\ r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}, show\ that\ \ x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y} + z\,\frac{\partial u}{\partial z} = 0$.

d) i) Change the order of integration in I = $\int_0^1 \int_{x^2}^{2-x} f(x,y)dy\,dx.$

ii) Prove that: β (m,n) = $\frac{\Gamma m \Gamma n}{\Gamma(m,+n)}$, m > 0 , n > 0.

e) i) Determine the value of constants a, b, c if, $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} +$ $(4x + cy + 2z)\hat{k}$ is irrotantional.

ii) If $\vec{A} = (x - y)\hat{\imath} + (x + y)\hat{\jmath}$, evaluate $\oint_c \vec{A}\,.\,\overrightarrow{dr}$ around the curve C consisting of $y = x^2$ and $y^2 = x.$

SECTION C

3. Attempt any *two* parts of the following: **7 x 5 = 35**

(a) *If* $y = e^{tan^{-1}} x$, *then prove that* $(1 + x^2)y_2 + (2x - 1)y_1 = 0$ and $(1 + x^2)\, y_{n+2} + [2(n + 1)x - 1]\, y_{n+1} + n(n + 1)y_n = 0$.

(b) *If* $u = x^2\, tan^{-1}\left(\frac{y}{x}\right) - y^2\, tan^{-1}\left(\frac{x}{y}\right)$; $xy \neq 0$ prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$.

(c) Trace the curve: $y^2(a + x) = x^2(3a - x)$.

4. Attempt any *two* parts of the following: **7 x 5 = 35**

(a) A balloon in the form of right circular of radius 1.5m and length 4m is surmounted by hemispherical ends. If the radius is increased by 0.01m find the percentage change in the volume of the balloon.

(b) Using Lagrange's method of Maxima and Minima, find the shortest distance from the point $(1,2,-1)$ to sphere $x^2 + y^2 + z^2 = 24$.

(c) Express the function $f(xy) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's Series expansion about the point (1, 2).

5. Attempt any *two* parts of the following: **7 x 5 = 35**

(a) Investigate for what values of λ and μ, the system of equations $x + y + z = 6, x + 2y + 3z = 10$ *and* $x + 2y + \lambda z = \mu$, *has*:
(i) No solution
(ii) Unique solution
(iii) Infinite number of solutions.

(b) Verify Clayey – Hamilton theorem for the matrices A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(c) Show that the matrix $\begin{bmatrix} \alpha + iy & -\beta + i\delta \\ \beta + i\delta & \alpha - iy \end{bmatrix}$ is unitary if $\alpha^2 + \beta^2 + y^2 + \delta^2 = 1$.

6. Attempt any *two* parts of the following: **7 x 5 = 35**

(a) Find the mass of a plate which is formed by the co-ordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, the density is given by $\rho =$ k xyz.

(b) Evaluate I = $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{½} dx$

(c) Evaluate $\iiint_R (x + y + z)\, dxdydz$ where R: $0 \leq x \leq 1$, $1 \leq y \leq 2$, $2 \leq z \leq 3$.

7. Attempt any *two* parts of the following: **7 x 5 = 35**

(a) Verify Green's theorem, evaluate $\int_c (x^2 + xy)\, dx + (x^2 + y^2)dy$ where c is square formed by lines $x = \pm 1$, $y = \pm 1$

(b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$.

(c) If all second order derivatives of ϕ and $\vec{v}$ are continuous, then show that Curl (gradϕ) = 0 (ii) div (curl $\vec{v}$) = 0.