

(SEM-I) THEORY EXAMINATION 2018-19
MATHEMATICS-I

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions.

Q no.	Question	Marks	CO
a.	Find the rank of the matrix $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$.	2	1
b.	Find the stationary point of $f(x, y) = x^3 + y^3 + 3axy, a > 0$	2	3
c.	If $x = r\cos\theta, y = r\sin\theta, z = z$ then find $\frac{\partial(r, \theta, z)}{\partial(x, y, z)}$	2	3
d.	Define del ∇ operator and gradient.	2	5
e.	If $\phi = 3x^2y - y^3z^2$, find grad ϕ at point (2, 0, -2).	2	4
f.	Evaluate $\int_0^1 \int_0^{x^2} e^x dy dx$	2	4
g.	If the eigen values of matrix A are 1, 1, 1, then find the eigen values of $A^2 + 2A + 3I$.	2	1
h.	Define Rolle's Theorem	2	2
i.	If $u = x^3y^2 \sin^{-1}(y/x)$, then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.	2	3
j.	In RI = E and possible error in E and I are 20% and 10% respectively, then find the error in R.	2	3
k.	State the Taylor's Theorem for two variables	2	3

SECTION B

2. Attempt any three of the following:

Q no.	Question	Marks	CO
a.	Using Cayley- Hamilton theorem find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Also express the polynomial $B = A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + 1$ as a quadratic polynomial in A and hence find B.	10	1

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b. If $y = \sin(m \sin^{-1}x)$, prove that: $(1 - x^2)y_{n+2} - (2n + 1)x y_n = 0$ and find y_n at $x = 0$.c. If u, v, w are the roots of the equation $(x - a)^3 + (x - b)^3 + (x - c)^3 = 0$, then find $\frac{\partial(u, v, w)}{\partial(a, b, c)}$.d. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2 + y^2)} dx dy$ by changing to polar coordinates.

$$\text{Hence show that } \int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

e. Verify the divergence theorem for

 $\vec{F} = (x^3 - yz)\hat{i} + (y^3 - zx)\hat{j} + (z^3 - xy)\hat{k}$, taken over the cube bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$.

SECTION C

3. Attempt any one part of the following:

Q no.

Question

a. Find inverse employing elementary transformation $A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ b. Reduce the matrix A to its normal form when $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 2 & -1 \end{bmatrix}$
Hence find the rank of A.

4. Attempt any one part of the following:

Q no.

Question

a. If $\sin^{-1} y = 2 \log(x+1)$ show that

$$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2 + 4)y_n = 0$$

b. Verify Lagrange's Mean value Theorem for the function $[-2, 2]$

5. Attempt any one part of the following:

Q no.

Question

a. Find the maximum or minimum distance of the point $(1, 2, 3)$ from the sphere $x^2 + y^2 + z^2 = 24$.b. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x+y}}\right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$



6. Attempt any one part of the following:

Q no.

Question

a.

$$\int_0^2 \int_{x^2}^{3-x} xy \, dy \, dx$$

- b. Calculate the volume of the solid bounded by the surface $x+y+z=1$ & $z=0$.

7. Attempt any one part of the following:

Q no.

Question

- a. Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is solenoidal and irrotational.

- b. Find the directional derivative of $\Phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at

P(1, 1, 1) in the direction of the line

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$