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(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID :199123 Roll No.

B.Tech.

(SEM. I) THEORY EXAMINATION, 2015-16

MATHEMATICS-I

[Time:3 hours] [Total Marks:100]

Section-A

1. Attempt all parts. All parts carry equal marks. Write answer of each part in shorts. (10×2=20)

(a) If $u = \log(x^2 / y)$ then value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$.

(b) If $z = xyf\left(\frac{x}{y}\right)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$.

(c) Apply Taylor's series find expansion of $f(x,y) = x^3 + xy^2$ about point (2,1), upto first degree term.

(d) If $x = u - v,\ y = u^2 - v^2$, find the value of $\frac{\partial(u,v)}{\partial(x,y)}$.

(e) Find all the asymptotes of the curve $xy^2 = 4a^2(2a-x)$.

(f) Find the inverse of the matrix by using elementary row operations. $A = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$

(g) If $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$, find the eigen values of A^2.

(h) Evaluate $\int_0^1 \int_1^2 \int_2^3 xyz \, dx \, dy \, dz$.

(i) If $\phi(x,y,z) = x^2y + y^2x + z^2$ find $\nabla\phi$ at the point $(1,1,1)$.

(j) Evaluate $\frac{\Gamma(8/3)}{\Gamma(2/3)}$.

Section-B

Note: Attempt any five Questions from this section:

(5x10=50)

2. If $x = \sin\left\{\frac{1}{m}\sin^{-1} y\right\}$ find the value of y_n at $x = 0$.

3. If u, v, w are the roots of the equation $(\lambda-x)^3 + (\lambda-y)^3 + (\lambda-z)^3 = 0$ in λ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

4. If r is the distance of a point on Conic $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = 0$ from origin, then that the stationary values of r are given by the equation
$$\frac{i^2}{1-ar^2} + \frac{m^2}{1-br^2} + \frac{n^2}{1-cr^2} = 0.$$

5. Find the Eigen values and corresponding Eigen vectors
$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

6. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. Also find its mass if the density at any point is $kxyz$.

7. Change the order of Integration in $I = \int_0^1 \int_{x^2}^{2-x} xy \, dxdy$ and hence evaluate the same.

8. Verify gauss's divergence theorem for the function $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$, taken over the cube bounded by $x=0, x=1, y=0, y=1$ and $z=1, z=1$.

9. Show that the Vector field $\vec{F} = \dfrac{\hat{r}}{|r|^3}$ is irrotational as well as solenoidal. Find the scalar potential.

Section-C

Attempt any two questions from this section: (2×15=30)

10. a) Expand $e^{ax}\cos by$ in powers of the powers of x and y as terms of third degree.

b) Determine the constant a and b such that the curl of vector.

$\bar{A} = (2xy+3xz)\,\hat{i} + (x^2 + axz - 4z^2)\hat{j} - (3xy + byz)\hat{k}$

is zero.

c) Examine the following vectors for linearly dependent and find the relation between them, if possible, $X_1 = (1,1-1,1)$, $X_2 = (1,-1,2-1)$, $X_3 = (3,1,0,1)$.

11. a) Define Beta and Gamma function and Evalaute $\int_0^1 \frac{dx}{\sqrt{1+x^4}}$.

b) Find the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

c) If $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_3x_1}{x_2}, y_3 \frac{x_1x_2}{x_3}$ find $\frac{\partial(y_1,y_2,y_3)}{\partial(x_1,x_2,x_3)}$.

12. a) Evaluate $\int_0^1 \frac{dx}{(a^n - x^n)^{1/n}}$

b) Reduce the matrix in to normal form and hence find its rank $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$

c) If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ show that

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$$

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