**B.TECH.**

**THEORY EXAMINATION (SEM–II) 2016-17**

**ENGINEERING MATHEMATICS - II**

*Time : 3 Hours* *Max. Marks : 100*

***Note :*** *Be precise in your answer. In case of numerical problem assume data wherever not provided.*

## SECTION – A

**1. Explain the following:** **10 x 2 = 20**

**(a)** Show that the differential equation y dx – 2x dy = 0 represents a family of parabolas.

**(b)** Classify the partial differential equation

$$(1-x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial y \partial x} + (1-y^2)\frac{\partial^2 z}{\partial y^2} = 2z$$

**(c)** Find the particular integral of $(D-\alpha)^2 y = e^{\alpha x} f''(x)$.

**(d)** Write the Dirichlet's conditions for Fourier series.

**(e)** Prove that $J'_0(x) = -J_1(x)$.

**(f)** Prove that $L\,[e^{at} f(t)] = F(s-a)$

**(g)** Find the Laplace transform of $f(t) = \frac{\sin at}{t}$.

**(h)** Write one and two dimensional wave equations.

**(i)** Find the constant term when $f(x) = |x|$ is expanded in Fourier series in the interval (-2, 2).

**(j)** Write the generating function for Legendre polynomial $P_n(x)$.

## SECTION – B

**2. Attempt any five of the following questions:** **5 x 10 = 50**

**(a)** Solve the differential equation

$(D^2 + 2D + 2)y = e^{-x} sec^3 x, \qquad where\ D = \frac{d}{dx}$.

**(b)** Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$, where $P_n(x)$ is the Legendre's function.

**(c)** Find the series solution of the differential equation

$$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x+1)y = 0.$$

**(d)** Using Laplace transform, solve the differential equation

$\frac{d^2y}{dt^2} + 9y = \cos 2t\ ;\quad y(0) = 1\ ,\ y\left(\frac{\pi}{2}\right) = -1.$

**(e)** Obtain the Fourier series of the function,

$f(t) = t \ , \quad -\pi < t < 0$

$\quad = -t \ , \quad 0 < t < \pi.$

Hence, deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$

**(f)** Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions $u(0, y) = 0$,

$$u(l, y) = 0, u(x, 0) = 0 \text{ and } u(x, a) = \sin\frac{n\pi x}{l}.$$

**(g)** Solve the partial differential equation:

$(D^3 - 4D^2D' + 5D\,D'^2 - 2D'^3)z = e^{y+2x} + \sqrt{y+x}$

**(h)** Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$

**SECTION – C**

**Attempt any two of the following questions:** **2 x 15 = 30**

**3.** **(a)** Solve the differential equation $(D^2-2D+1)y = e^x \sin x$

**(b)** Solve the equation by Laplace transform method:

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t\,, \quad y(0) = 1.$$

**(c)** Solve the partial differential equation

$$(y^2 + z^2)\,p - xyq + zx = 0, where\ p = \frac{\partial z}{\partial x}\ \&\ q = \frac{\partial z}{\partial y}$$

**4.** **(a)** Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.

**(b)** Express $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre's polynomial.

**(c)** Expand $f(x) = 2x - 1$ as a cosine series in $0 < x < 2$.

**5.** **(a)** Show that $J_3(x) = \left(\frac{8}{x^2} - 1\right) J_1(x) - \frac{4}{x} J_0(x)$.

**(b)** Solve the $2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} + 5z = 0;\ z(0, y) = 2e^{-y}$ by the method of separation of variables.

**(c)** A tightly stretched string with fixed end $x = 0$ and $x = l$ is initially in a position given by $y = a \sin\frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x, t)$.