B.TECH.

THEORY EXAMINATION (SEM–II) 2016-17

ENGINEERING MATHEMATICS - II

Time : 3 Hours ***Max. Marks : 100***

Note : *Be precise in your answer. In case of numerical problem assume data wherever not provided.*

SECTION – A

1. Explain the following: **10 x 2 = 20**

(a) Show that the differential equation y dx – 2x dy = 0 represents a family of parabolas.

(b) Classify the partial differential equation

$$(1-x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial y \partial x} + (1-y^2)\frac{\partial^2 z}{\partial y^2} = 2z$$

(c) Find the particular integral of $(D-\alpha)^2 y = e^{\alpha x} f''(x)$.

(d) Write the Dirichlet's conditions for Fourier series.

(e) Prove that $J'_0(x) = -J_1(x)$.

(f) Prove that $L\,[e^{at} f(t)] = F(s-a)$

(g) Find the Laplace transform of $f(t) = \frac{\sin at}{t}$.

(h) Write one and two dimensional wave equations.

(i) Find the constant term when $f(x) = |x|$ is expanded in Fourier series in the interval (-2, 2).

(j) Write the generating function for Legendre polynomial $P_n(x)$.

SECTION – B

2. Attempt any five of the following questions: **5 x 10 = 50**

(a) Solve the differential equation

$(D^2 + 2D + 2)y = e^{-x} sec^3 x, \quad where\ D = \frac{d}{dx}$.

(b) Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$, where $P_n(x)$ is the Legendre's function.

(c) Find the series solution of the differential equation

$$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x+1)y = 0.$$

(d) Using Laplace transform, solve the differential equation

$\frac{d^2y}{dt^2} + 9y = \cos 2t\ ;\ \ y(0) = 1\ ,\ y\left(\frac{\pi}{2}\right) = -1.$

(e) Obtain the Fourier series of the function,

$f(t) = t \ , \ -\pi < t < 0$

$= -t \ , \ 0 < t < \pi.$

Hence, deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots = \frac{\pi^2}{8}$

(f) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions $u(0, y) = 0$,

$$u(l, y) = 0, u(x, 0) = 0 \text{ and } u(x, a) = \sin\frac{n\pi x}{l}.$$

(g) Solve the partial differential equation:

$(D^3 - 4D^2D' + 5D\,D'^2 - 2D'^3)z = e^{y+2x} + \sqrt{y+x}$

(h) Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$

SECTION – C

Attempt any two of the following questions: **2 x 15 = 30**

3. **(a)** Solve the differential equation $(D^2-2D+1)y = e^x \sin x$

(b) Solve the equation by Laplace transform method:

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t\,, \quad y(0) = 1.$$

(c) Solve the partial differential equation

$$(y^2 + z^2)\, p - xyq + zx = 0, where\ p = \frac{\partial z}{\partial x}\ \&\ q = \frac{\partial z}{\partial y}$$

4. **(a)** Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.

(b) Express $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre's polynomial.

(c) Expand $f(x) = 2x - 1$ as a cosine series in $0 < x < 2$.

5. **(a)** Show that $J_3(x) = \left(\frac{8}{x^2} - 1\right) J_1(x) - \frac{4}{x} J_0(x)$.

(b) Solve the $2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} + 5z = 0;\ z(0, y) = 2e^{-y}$ by the method of separation of variables.

(c) A tightly stretched string with fixed end $x = 0$ and $x = l$ is initially in a position given by $y = a \sin\frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x, t)$.