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B. TECH
(SEM-II) THEORY EXAMINATION 2017-18
ENGINEERING MATHEMATICS - II

Time: 3 Hours ***Total Marks: 70***

Note: Attempt all Sections. If require any missing data, then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 7 = 14**

(a) Determine the differential equation whose set of independent solutions is $\{e^x, xe^x, x^2e^x\}$.

(b) Solve: $(D+1)^3 y = 2e^{-x}$.

(c) Prove that: $P_n(-x) = (-1)^n P_n(x)$.

(d) Find inverse Laplace transform of $\frac{s+8}{s^2+4s+5}$.

(e) If $L\{F(\sqrt{t})\} = \frac{e^{-1/s}}{s}$, find $L\{e^{-t}F(3\sqrt{t})\}$.

(f) Solve: $(D+4D'+5)^2 z = 0$, where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$.

(g) Classify the equation: $z_{xx} + 2xz_{xy} + (1-y^2)z_{yy} = 0$.

SECTION B

2. Attempt any *three* of the following: **7 x 3 = 21**

(a) Solve $(D^2 - 2D + 4)y = e^x \cos x + \sin x \cos 3x$.

(b) Prove that: $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\left(\frac{3-x^2}{x^2}\right)\sin x - \frac{3\cos x}{x}\right]$.

(c) Draw the graph and find the Laplace transform of the triangular wave function of period 2π given by

$$F(t) = \begin{cases} t, & 0 < t \le \pi \\ 2\pi - t, & \pi < t < 2\pi \end{cases}.$$

(d) Obtain half range cosine series for e^x the function $f(t) = \begin{cases} 2t, & 0 < t < 1 \\ 2(2-t), & 1 < t < 2 \end{cases}$

(e) Solve by method of separation of variables: $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} - 2u$; $u(x,0) = 10e^{-x} - 6e^{-4x}$.

SECTION C

3. Attempt any *one* part of the following: **7 x 1 = 7**

(a) Solve the simultaneous differential equations:

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = y \text{ and } \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 25x + 16e^t.$$

(b) Use variation of parameter method to solve the differential equation $x^2y'' + xy' - y = x^2e^x$.

4. Attempt any *one* part of the following: 7 x 1 = 7

(a) State and prove Rodrigue's formula for Legendre's polynomial.

(b) Solve in series: $2x(1-x)y'' + (1-x)y' + 3y = 0$.

5. Attempt any *one* part of the following: 7 x 1 = 7

(a) State convolution theorem and hence find inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.

(b) Solve the following differential equation using Laplace transform $\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = t^2e^t$ where $y(0)=1$, $y'(0)=0$ and $y''(0)=-2$.

6. Attempt any *one* part of the following: 7 x 1 = 7

(a) Obtain Fourier series for the function $f(x) = \begin{cases} x, & -\pi < x \le 0 \\ -x, & 0 < x < \pi \end{cases}$ and hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.

(b) Solve the linear partial differential equation: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$.

7. Attempt any *one* part of the following: 7 x 1 = 7

(a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = A\sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement of any point at a distance x from one end at any time t.

(b) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along one short edge $y = 0$ is given by $u(x,0) = 100\sin\frac{\pi x}{8}$, $0 < x < 8$ while the two long edges $x = 0$ and $x = 8$ as well as the other short edge are kept at $0^\circ C$. Find the temperature $u(x, y)$ at any point in steady state.