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AS-303

(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID :199312

Roll No.

B.Tech.

(SEM. III) THEORY EXAMINATION, 2015-16

MATHEMATICS-III

[Time:3 hours]

[Maximum Marks:100]

Note: Attempt all questions from each Section as indicated.
The symbols have their usual meaning.

Section-A

1. Attempt all parts of this section. Each part carry 2 marks.

(2×10=20)

- (a) Show that $w=iz$ is the rotation of the z -plane through an angle $\pi/2$ in the counterclockwise direction.

- (b) Determine and classify all the singularity of

$$\frac{1}{z(z-2)^3} + \frac{1}{(z-2)^2}$$

3800

(1)

P.T.O.

- (c) Define Fourier Transform of a function $f(x)$.
- (d) Find the Z-Transform of $\{(-1)^n\}$.
- (e) Define Probability density function.
- (f) What is Karl Pearson's coefficient of skewness.
- (g) Show that $\nabla \cdot \Delta = -\nabla \Delta$.
- (h) Define Bisection method.
- (i) What is cubic spline?
- (j) Find missing value in following table:

X	45	50	55	60	64
Y	3	-	2	-	-2.4

Section-B

Attempt any five questions from this section. (5×10=50)

2. (a) Show that the function defined by $f(x) = \sqrt{|xy|}$ is not regular at origin, although Cauchy-Riemann equations are satisfied.
- (b) Determine the analytic function $f(z) = u + iv$, in terms of z , whose $u - v = e^x (\cos y - \sin y)$.

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(2)

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3. (a) Find inverse Z-Transform of $\frac{1}{(z-5)^3}$, when $z > 5$
- (b) Solve the following difference equation using Z-transform $u_{n+2} + 2u_{n+1} + u_n = n, u_0 = u_1 = 0$.
4. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that if $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$, when of $(0.5)=0.19$, and $f(1.4)=0.42$.

- (b) In a bombing action, there is a 50% chance that any bomb will strike the target. Two direct hits are needed to destroy the target completely. How many bombs are required to be dropped to give a 99% chance of better of completely destroying the target.

5. (a) Find to four places of decimal, the smallest root the equation $e^{-x} = \sin x$.

- (b) From the following table find the value of $e^{0.24}$.

X	0.1	0.2	0.3	0.4	0.5
Y	1.10517	1.2214	1.34986	1.49182	1.64872

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(3)

P.T.O.

6. (a) The distance covered an athlete for the 50 meter race is given as:

Time (sec)	0	1	2	3	4	5	6
Distance (meter)	0	2.5	8.5	15.5	24.5	36.5	50

Determine speed of the athlete at $t=5$ sec correct to two decimal.

- (b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $3/8^{\text{th}}$ rule, by taking $h=1/6$.

7. (a) Evaluate using Cauchy integral formula.

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz, \text{ where } C \text{ is the circle } |z|=4.$$

- (b) Find the Fourier Sine transform of :

$$f(x) = e^{-ax}, \text{ for } x \geq 0 \text{ and } a > 0.$$

hence show that,

$$\int_0^\infty \frac{a \sin ax}{a^2 + a'^2} da = \frac{\pi e^{-ax}}{2}$$

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(4)

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8. (a) Six coins are tossed 6400 times. Using the Poisson distribution, determine the probability of getting six heads x times.

- (b) Using Newton's divided difference formula find a polynomial which takes the values 3, 12, 15, -21 when x has the values 3, 2, 1 and -1 respectively.

9. (a) using Milne's method, solve $\frac{dy}{dx} = 1+y^2$ with initial conditions.

$$y(0)=0, y(0.2)=0.2027, y(0.4)=0.4228, y(0.6)=0.6841, \text{ find } y(0.8).$$

- (b) Find the value of $y(0.6)$ by Runga Kutta fourth order method taking $h=0.2$ for the initial value problem..

Section-C

10. Attempt any two parts of this Section. (15x2=30)

- (a) Apply calculus of residues to evaluate.

$$\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx, a > 0.$$

3800

(5)

P.T.O

- (b) Solve the equation. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}, x > 0, t > 0$

Subject to the conditions:

- (i) $y = 0$ when $x = 0$, (ii) $f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, x > 1 \end{cases}$ (iii) $u(x, t)$ is bounded.
- (c) The first four moments about working mean 28.5 of a distribution are 0.294, 7.144, 42.409, and 454.98. Calculate the moments about mean. Also calculate β_1 and β_2 and comment upon the skewness and kurtosis of the distribution.
- (d) Use Gauss-Seidal method to solve the following equations,

$$2x + 10y + z = 51$$

$$10x + y + 2z = 44$$

$$x + 2y + 10z = 61$$

—x—