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(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID :199312 Roll No. | | | | | | | | | | |

**B.Tech.**

**(SEM. III) THEORY EXAMINATION, 2015-16**

**MATHEMATICS-III**

**[Time:3 hours]** **[MaximumMarks:100]**

**Note:** Attempt **all** questions from each Section as indicated. The symbols have their usual meaning.

**Section-A**

1. Attempt **all** parts of this section. Each part carry 2 marks. (2×10=20)

(a) Show that w=iz is the rotation of the z-plane through an angle $\pi/2$ in the counterclockwise direction.

(b) Determine and classify all the singularity of $\frac{1}{z(z-2)^5}+\frac{1}{(z-2)^2}$.

(c) Define Fourier Transform of a function f(x).

(d) Find the Z- Transform of $\{(-1)^n\}$.

(e) Define Probability density function.

(f) What is Karl Pearson's coefficient of skewness.

(g) Show that $\nabla - \Delta = -\nabla\Delta$.

(h) Define Bisection method.

(i) What is cubic spline?

(j) Find missing value in following table:

| X | 45 | 50 | 55 | 60 | 64 |
|---|---|---|---|---|---|
| Y | 3 | - | 2 | - | -2.4 |

**Section-B**

Attempt **any five** questions from this section. (5×10=50)

2. (a) Show that the function defined by $f(x) = \sqrt{|xy|}$ is not regular at origin, although Cauchy-Riemann equations are satisfied.

(b) Determine the analytic function f(z) =u+iv, in terms of z, whose $u - v = e^x(\cos y - \sin y)$.



3. (a) Find inverse Z-Transform of $\frac{1}{(z-5)^{-3}}$, when $z > 5$

(b) Solve the following difference equation using Z-transform $u_{n+2} + 2u_{n+1} + u_n = n, u_0 = u_1 = 0$.

4. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that if $f(t) = \frac{1}{\sqrt{2\pi}}\int_0^t e - \frac{1}{2}x^2 dx$, when of (0.5)=0.19, and f (1.4)=0.42.

(b) In a bombing action, there is a 50% chance that any bomb will strike the target. Two direct hits are needed to destory the targey completely. How many bombs are required to be dropped to give a 99% chance of better of completely destroying the target.

5. (a) Find to four places of decimal, the smallest root the equation $e^{-x} = \sin x$.

(b) From the following table find the value of $e^{0.24}$.

| X | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|
| Y | 1.10517 | 1.2214 | 1.34986 | 1.49182 | 1.64872 |

6. (a) The distance covered an athlete for the 50 meter race is given as:

| Time (sec) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Distance (meter) | 0 | 2.5 | 8.5 | 15.5 | 24.5 | 36.5 | 50 |

Determine speed of the athlete at t=5 sec correct to two decimal.

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's 3/8th rule, by taking h=1/6.

7. (a) Evaluate using Cauchy intergral formula.

$\oint_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}$ dz, where C is the circle |z| =4.

(b) Find the Fourier Sine transform of :

$f(x) = e^{-ax}$, for x $\geq$ 0 and a > 0.

hence show that,

$$\int_0^\infty \frac{a\sin ax}{a^2+a^2}da = \frac{\pi e^{-ax}}{2}$$



8. (a) Six coins are tossed 6400 times. Using the Poisson distribution, determine the probability of getting six heads x times.

(b) Using Newton's divided difference formula find a polynomial which takes the values 3, 12, 15, -21 when x has the values 3, 2, 1 and -1 respectively.

9. (a) using Milne's method, solve $\frac{dy}{dx} = 1+y^2$ with initial conditions.

y(0)=0, y(0.2)=0.2027, y(0.4)= 0.4228,y (0.6)=0.6841, find y(0.8).

(b) Find the value of y (0.6) by Ranga Kutta fourth order method taking h=0.2 for the initial value problem:.

## Section-C

10. Attempt any two parts of this Section. (15x2=30)

(a) Apply calculus of residues to evaluate.

$$\int_0^\infty \frac{x\sin x}{x^2+a^2}dx, a>0.$$

(b) Solve the equation. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}, x > 0, t > 0$

Subject to the conditions:

(i) $y = 0$ when $x = 0$, (ii) $f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, x > 1 \end{cases}$ (iii) u (x,t) is bounded.

(c) The first four moments about working mean 28.5 of a distribution are 0.294, 7.144, 42.409, and 454.98. Calculate the moments about mean. Also calculate $\beta_1$ and $\beta_2$ and comment upon the skewness and kurtosis of the distribution.

(d) Use Gauss-Seidal method to solve the following equations,

$$2x + 10y + z = 51$$
$$10x + y + 2z = 44$$
$$x + 2y + 10z = 61$$

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