

**B.TECH.**

**THEORY EXAMINATION (SEM-IV) 2016-17**

**MATHEMATICS-II**

**Time : 3 Hours**

**Max. Marks : 70**

**Note : Be precise in your answer. In case of numerical problem assume data wherever not provided.**

**SECTION – A**

**1. Attempt any seven parts for the following:**

**7 x 2 = 14**

- Solve the differential equation  $\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$  with the condition  $x = 0, y = 5$  and  $x = 0, y = 21$  and hence find the value of  $y$  at  $x = 1$ .
- For a differential equation  $\frac{d^2y}{dx^2} + 2\alpha\frac{dy}{dx} + y = 0$ , find the value of  $\alpha$  for which the differential equation characteristic equation has equal number.
- For a Legend polynomial prove that  $P_n(1) = 1$  and  $P_n(-1) = (-1)^n$
- For the Bessel's function  $J_n(x)$  prove the following identities:  
 $J_{-n}(x) = (-1)^n J_n(x)$  and  $J_{-n}(-x) = (-1)^n J_n(x)$
- Evaluate the Laplace transform of Integral of a function  $L\left\{\int_0^t f(t/dt)\right\}$ .
- Evaluate the value of integral  $\int_0^\infty t \cdot e^{-2t} \cos t \, dt$ .
- Find the Fourier coefficient for the function  $f(x) = x^2 \quad 0 < x < 2\pi$
- Find the partial differential equation of all sphere whose centre lie on Z-axis.
- Formulate the PDE by eliminating the arbitrary function from  $\phi(x^2 + y^2, y^2 + z^2) = 0$
- Specify with suitable example the clarification Partial Differential Equation (PDE) for elliptic, parabolic and hyperbolic differential equation.

**SECTION – B**

**2. Attempt any three parts of the following questions:**

**3 x 7 = 21**

- A function  $n(x)$  satisfies the differential equation  $\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$ , where  $L$  is a constant. The boundary conditions are  $n(0) = x$  and  $n(\infty) = 0$ . Find the solution to this equation.
- Find the series solution by Forbenias method for the differential equation  
 $(1 - x^2)y'' - 2xy' + 20y = 0$
- Determine the response of damped mass – spring system under a square wave given by the differential equation  
 $y'' + 3y' + 2y = u(t - 1) - u(t - 2), \quad y(0) = 0, \quad y'(0) = 0$   
Using the Laplace transform.
- Obtain the Fourier expansion of  $f(x) = x \sin x$  as cosine series in  $(0, \pi)$  and hence show that  
$$\frac{1}{1 \times 3} - \frac{1}{3 \times 5} + \frac{1}{5 \times 7} - \dots = \left(\frac{\pi - 2}{4}\right)$$
- Solve by method of separation of variable for PDE  
 $x \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0, \quad u(x, 0) = 4e^{-x}$

**Attempt all parts of the following questions:**

3. Attempt any two parts of the following:

(a) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + a^1 = \sec ax$$

(b) If  $y = y_1(x)$  and  $y = y_2(x)$  are two solutions of the equation  $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ , then show that  $y_1\left(\frac{dy_2}{dx}\right) - y_2\left(\frac{dy_1}{dx}\right) = ce^{-\int P dx}$ , where  $c$  is constant.

(c) Solve by method of variation of Parameter for the differential equation :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ay = \left(\frac{e^{3x}}{x^2}\right)$$

4. Attempt any two parts of the following:

(a) Prove that  $\sqrt{\frac{\pi x}{2}} \cdot J_{3/2}(x) = \left(\frac{1}{x} \sin x - \cos x\right)$

(b) Show that Legendre polynomials are orthogonal on the interval  $[-1, 1]$

(c) Prove that  $\int_{-1}^{+1} x P_n(x) dx = \frac{2n}{4n^2 - 1}$

5. Attempt any two parts of the following:

(a) Find the Laplace transform of  $S_{RW}$  – tooth wave function

$F(t) = Kt$  in  $0 < t < 1$  with period 1

(b) Use Convolution theorem to find the inverse of function  $F(s) = \frac{4}{s^2 + 2s + 5}$

(c) Solve the simultaneous differential equation, using Laplace transformation –  
 $\frac{dy}{dt} + 2x = \sin 2t$ ;  $\frac{dy}{dt} - 2y = \cos 2t$ , where  $x(0) = 1$ ,  $y(0) = 0$

6. Attempt any two parts of the following:

(a) If  $f(x) = \left[\frac{\pi - x}{2}\right]^2$ ,  $0 < x < 2\pi$  then show that  $f(x) = \frac{\pi^2}{12} - \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$

(b) Find the complete solution of PDE

$(\Delta^2 + 7\Delta D' + 12D'^2)/2 = \sin hx$ , where symbols have their usual meaning.

(c) Solve the PDE  $p + 3q = 5z + \tan(y - 3x)$

7. Attempt any one part of the following:

(a) A square plate is bounded by lines  $x = 0$ ,  $y = 0$ ;  $x = 20$ ,  $y = 20$ . Its faces are insulated. The temperature along the upper horizontal edge is given by  $u(x, 20) = x(20 - x)$  when  $0 < x < 20$  while the upper three edges are kept at  $0^\circ\text{C}$ . Find the steady state temperature.

(b) A bar of 10 cm long with insulated sides A and B are kept at  $20^\circ\text{C}$  and  $40^\circ\text{C}$  respectively until steady state conditions prevail. The temperature at A is then suddenly varies to  $50^\circ\text{C}$  and the same instant that at B lowered to  $10^\circ\text{C}$ . Find the subsequent temperature at any point of the bar at any time.