

B. TECH.

THEORY EXAMINATION (SEM-IV) 2016-17
ENGINEERING MATHEMATICS-III

Time : 3 Hours

Max. Marks : 100

SECTION – A

1. Attempt all parts of the following question: (2 x 10 = 20)

- (a) Evaluate $\int_C \frac{e^z}{z+1} dz$, where C is the circle $|z| = 2$
- (b) Prove that $f(z) = \sinh z$ is analytic
- (c) Prove that Modulation theorem $F\{f(x) \cos ax\} = \frac{1}{2}[f(s+a) + f(s-a)]$
- (d) Solve the Z-transform: $y_{k+2} + y_{k+1} - 2y_k = 0$, $y_0 = 4$, $y_1 = 0$
- (e) What is the meaning of Skewness?
- (f) Write Normal equation of $y = a + \frac{b}{x}$
- (g) Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$
- (h) Find first approximation value of $(17)^{1/3}$ by using Newton Raphson method
- (i) Using Picard's method, find the solution of $\frac{dy}{dx} = 1 + xy$ upto the third approximation when $x(0) = 0$
- (j) Find $y(0.1)$ using Euler's method given that $\frac{dy}{dx} = \log(x+y)$, $y(0) = 1.0$

SECTION – B

2. Attempt any five parts of the following question: (5 x 10 = 50)

- (a) Prove that the function $f(z)$ defined by $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ is continuous and the C.R. equations are satisfied at the origin, yet $f'(0)$ does not exist.
- (b) Using Cauchy Integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 3$.
- (c) Find the Fourier cosine transform of $\frac{1}{1+x^2}$ and then find Fourier sine transform of $\frac{x}{1+x^2}$.
- (d) Find the multiple linear regression of X_1 on X_2 and X_3 from the data relating to three variables:

X_1	7	12	17	20
X_2	4	7	9	12
X_3	1	2	5	8

- (e) Find the root of the equation $x^3 + 3x^2 + 2x + 1 = 0$ by using Regular-Fabius method correct to four decimal places.

$$2x + 3y + z = 9$$

- (f) Apply Crout's method and solve the system of equations $x + 2y + 3z = 6$

$$3x + y + 2z = 8$$

- (g) Find the value $y(1.1)$ using Runge-Kutta method of fourth order, given that

$$\frac{dy}{dx} = y^2 + xy, y(1) = 1.0, \text{ take } h = 0.05$$

SECTION – C

Attempt any two questions of the following:

(2 x 15 = 30)

3. (i) Show that the function defined by $f(z) = \sqrt{|xy|}$ is not regular at the origin, although Cauchy-Riemann equations are satisfied

(ii) Evaluate: $\int_0^{2\pi} \frac{d\theta}{a + b \sin \theta}$ if $a > |b|$

(iii) Solve by Z-transform: $y_{k+2} - 4y_{k+1} + 3y_k = 5^k$

4. (i) Using the convolution theorem, evaluate $Z^{-1} \left\{ \frac{z^2}{(z-1)(z-3)} \right\}$

- (ii) If the θ is the acute angle between the two regression lines in the case of two variables x and y , show that $\tan \theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$, where r, σ_x, σ_y have their usual meanings. Explain the significance of the formula when $r = 0$ and $r = \pm 1$

- (iii) By using χ^2 -test, find out whether there is any association between income level and type of schooling :

Social status Health	Poor	Rich	Total
Below Normal	130	20	150
Normal	102	108	210
Above Normal	24	96	120
Total	256	224	480

5. (i) Find the missing figure in the following table

x	2	3	4	5	6
$f(x)$	45	49.2	54.1	?	67.4

- (ii) Find a cubic polynomial which approximates the data:

x	-2	-1	2	3
$y(x)$	-12	-8	3	5

- (iii) Find an approximate value of the $\log_e 5$ by calculating to four decimal places

by Simpson's $\frac{1}{3}$ rule, given $\int_0^5 \frac{dx}{4x+5}$