

B. TECH.
(SEM-IV) THEORY EXAMINATION 2017-18
MATHEMATICS - III

Time: 3 Hours**Total Marks: 70****Note:** Attempt all Sections. If require any missing data, then choose suitably.

SECTION A

1. Attempt all questions in brief. 2 x 7 = 14

- a) Discuss Singularity and its types.
- b) Write Cauchy-Riemann equation in polar co-ordinates.
- c) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are insured, how many pairs would be expected to need replacement after 12 months? Given that $P(z \geq 2) = 0.0228$.
- d) Determine moment generating function of Binomial distribution.
- e) Prove that: $E^{\frac{1}{2}} = \mu + \frac{1}{2}\delta$
- f) Write Newton-Cote's quadrature formula.
- g) Find Z-transform of $f(k) = u(-k)$.

SECTION B

2. Attempt any three of the following: 7 x 3 = 21

- a) Determine an analytic function $f(z)$ in terms of z if $u + v = 2 \frac{\sin 2x}{e^{2y}} + e^{2y} - 2 \cos 2x$.
 - b) Find the mean variance of Poisson distribution.
 - c) Find $\int_0^6 \frac{e^x}{1+x} dx$ using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule and (iii) Simpson's 3/8th rule.
 - d) A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of time t (in seconds).
- | | | | | | | | |
|-----------|---|------|------|------|------|------|------|
| $t:$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 |
| $\theta:$ | 0 | 0.12 | 0.49 | 1.12 | 2.02 | 3.20 | 4.67 |

Calculate the angular velocity and angular acceleration at $t = 0.2$ and $t = 1.2$ second.

- e) Find Fourier cosine transform of $\frac{1}{1+x^2}$, hence find Fourier sine transform of $\frac{1}{1+x^2}$

3. Attempt any one part of the following:
7 x 1 = 7

- (a) Verify Cauchy theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i$, $-1+i$ and $-1-i$.
- (b) Evaluate $\int_0^\infty \frac{\sin mx}{x} dx$, $m > 0$.

4. Attempt any one part of the following:
7 x 1 = 7

- (a) The following table represents the height of a batch of 100 students. Calculate skewness and kurtosis:

Height (in cm)	59	61	63	65	67	69	71	73	75
No. of students	0	2	6	20	40	20	8	2	2

- (b) Use the method of least squares to fit the curve $y = \frac{c_0}{x} + c_1\sqrt{x}$ to the following table of values:

x	0.1	0.2	0.4	0.5	1	2
y	21	11	7	6	5	6

5. Attempt any one part of the following:
7 x 1 = 7

- (a) Find the root of the equation $xe^x = \cos x$ using Regula-Falsi method correct to four decimal places.
- (b) Find Newton's divided difference polynomial for the following data:
- | | | | | | |
|-------|-----|-----|-----|-----|------|
| x: | -3 | -1 | 0 | 3 | 5 |
| f(x): | -30 | -22 | -12 | 330 | 3458 |

6. Attempt any one part of the following:
7 x 1 = 7

- (a) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$. Use Runge-Kutta fourth order method and compare your result with exact solution.
- (b) Solve the following system of linear equations by Matrix decomposition method taking $l_{ii} = 1$ for $1 \leq i \leq 3$.

$$3x - y + 2z = 12; \quad x + 2y + 3z = 11; \quad 2x - 2y - z = 2$$

7. Attempt any one part of the following:
7 x 1 = 7

- (a) Using Z-transform, solve the following difference equation:

$$y_{k+2} + 4y_{k+1} + 3y_k = 3^k, \text{ given that } y_0 = 0 \text{ and } y_1 = 1.$$

- (b) The temperature u in the semi-infinite rod $0 \leq x < \infty$ is determined by the differential equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \text{ subject to conditions}$$

- (i) $u = 0$ when $t = 0, x \geq 0$ (ii) $\frac{\partial u}{\partial x} = -\mu$ (a constant) when $x = 0$ and $t > 0$, (iii) $u(x, t)$ is bounded.

Determine the temperature $u(x, t)$.