DU MA MSc Mathematics

Topic:- DU_J18_MA_MATHS_Topic01

1)
The complete integral of the partial differential equation $xpq + yq^2 - 1 = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ is

[Question ID = 2159]

1. $(z+b)^2 = 4(ax+y)$. [Option ID = 8635]
2. $z+b = 2(ax+y)$. [Option ID = 8633]
3. $z+b = 4(ax+y)^2$. [Option ID = 8636]
4. $z+b = 2(ax+y)^2$. [Option ID = 8634]

Correct Answer :-

- $(z+b)^2 = 4(ax+y)$. [Option ID = 8635]

2)
Let P be the set of all the polynomials with rational coefficients and S be the set of all sequences of natural numbers. Then which one of the following statements is true?

[Question ID = 2139]

1. S is countable but P is not. [Option ID = 8555]
2. Both the sets P and S are uncountable. [Option ID = 8556]
3. Both the sets P and S are countable. [Option ID = 8553]
4. P is countable but S is not. [Option ID = 8554]

Correct Answer :-

- P is countable but S is not. [Option ID = 8554]

3)
For the differential equation

$$x\frac{dy}{dx} + 6y = 3xy^{4/3}$$

consider the following statements:

(i) The given differential equation is a linear equation.

(ii) The differential equation can be reduced to linear equation by the transformation $V = y^{-1/3}$.

(iii) The differential equation can be reduced to linear equation by the transformation $V = x^{-1/3}$.

Which of the above statements are true?

are [Question ID = 2156]

1. Only (i). [Option ID = 8622]
2. Only (iii). [Option ID = 8624]
3. Only (ii). [Option ID = 8623]
4. Both (i) and (ii). [Option ID = 8621]

Correct Answer :-

- Only (ii). [Option ID = 8623]

4)

Which one of the following statements is not true for Simpson's 1/3 rule to find approximate value of the definite integral $I = \int_0^1 f(x)dx$?

[Question ID = 2151]

1. If $y_0 = f(0), y_1 = f(0.5), y_2 = f(1)$, the approximate value of I is $\frac{1}{6}[y_0 + 3y_1 + y_2]$. [Option ID = 8603]
2. The approximating function has odd number of points common with the function $f(x)$. [Option ID = 8604]
3. Simpson's 1/3 rule improves trapezoidal rule. [Option ID = 8602]
4. The function $f(x)$ is approximated by a parabola. [Option ID = 8601]

Correct Answer :-

- If $y_0 = f(0), y_1 = f(0.5), y_2 = f(1)$, the approximate value of I is $\frac{1}{6}[y_0 + 3y_1 + y_2]$. [Option ID = 8603]

5) The equation of the tangent plane to the surface $z = 2x^2 - y^2$ at the point (1, 1, 1) is

[Question ID = 2133]

1. $x - y - 2z = 2.$ [Option ID = 8531]
2. $4x - y - 3z = 1.$ [Option ID = 8532]
3. $2x - y - 2z = 1.$ [Option ID = 8529]
4. $4x - 2y - z = 1.$ [Option ID = 8530]

Correct Answer :-

- $4x - 2y - z = 1.$ [Option ID = 8530]

6)

If $\{x, y\}$ is an orthonormal set in an inner product space then the value of $\|x - y\| + \|x + y\|$ is

[Question ID = 2128]

1. $2\sqrt{2}.$ [Option ID = 8510]
2. $2 + \sqrt{2}.$ [Option ID = 8512]
3. $\sqrt{2}.$ [Option ID = 8511]

4. 2. [Option ID = 8509]

Correct Answer :-

- $2\sqrt{2}$. [Option ID = 8510]

7) Which one of the following spaces, with the usual metric, is not separable?

[Question ID = 2147]

1. The space $C[a, b]$ of the set of all real valued continuous functions defined on $[a, b]$. [Option ID = 8586]
2. The space l^∞ of all bounded real sequences with supremum metric. [Option ID = 8588]
3. The Euclidean space $\mathbb{R}^n$. [Option ID = 8585]
4. The space l^1 of all absolutely convergent real sequences. [Option ID = 8587]

Correct Answer :-

- The space l^∞ of all bounded real sequences with supremum metric. [Option ID = 8588]

8) Let G be an abelian group of order 2018 and $f: G \to G$ be defined as $f(x) = x^5$. Then

[Question ID = 2118]

1. f is not injective. [Option ID = 8470]
2. f is not surjective. [Option ID = 8471]
3. there exists $e \neq x \in G$ such that $f(x) = x^{-1}$. [Option ID = 8472]
4. f is an automorphism of G. [Option ID = 8469]

Correct Answer :-

- f is an automorphism of G. [Option ID = 8469]

9) If $f: \mathbb{R} \to \mathbb{R}$ is a continuous function such that

$$f(x+y) = f(x) + f(y), \text{ for all } x, y \in \mathbb{R},$$

then

[Question ID = 2138]

1. f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$. [Option ID = 8551]
2. f is increasing if $f(1) \leq 0$ and decreasing if $f(1) \geq 0$. [Option ID = 8552]
3. f is a not an increasing function. [Option ID = 8549]
4. f is neither an increasing nor a decreasing function. [Option ID = 8550]

Correct Answer :-

- f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$. [Option ID = 8551]

10) The central difference operator δ and backward difference operator ∇ are related as

[Question ID = 2154]

1. $\delta = \nabla(1-\nabla)^{\frac{1}{2}}.$ [Option ID = 8615]
2. $\delta = \nabla(1+\nabla)^{-\frac{1}{2}}.$ [Option ID = 8614]
3. $\delta = \nabla(1-\nabla)^{-\frac{1}{2}}.$ [Option ID = 8616]
4. $\delta = \nabla(1+\nabla)^{\frac{1}{2}}.$ [Option ID = 8613]

Correct Answer :-

- $\delta = \nabla(1-\nabla)^{-\frac{1}{2}}.$ [Option ID = 8616]

11) How many continuous real functions f can be defined on $\mathbb{R}$ such that $(f(x))^2 = x^2$ for every $x \in \mathbb{R}$?

[Question ID = 2144]

1. Infinitely many. [Option ID = 8576]
2. None. [Option ID = 8575]
3. 4. [Option ID = 8574]
4. 2. [Option ID = 8573]

Correct Answer :-

- 4. [Option ID = 8574]

12) The greatest common divisor of $11+7i$ and $18-i$ in the ring of Gaussian integers $\mathbb{Z}[i]$ is

[Question ID = 2122]

1. $3i.$ [Option ID = 8485]
2. $1.$ [Option ID = 8488]
3. $1+i.$ [Option ID = 8487]
4. $2+i.$ [Option ID = 8486]

Correct Answer :-

- $1.$ [Option ID = 8488]

13) The complete integral of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$$

is

[Question ID = 2161]

1. $\phi_1(y-x) + x\phi_2(y+x) + e^{x+2y}.$ [Option ID = 8643]

2. $\phi_1(y+x) + x\phi_2(y+x) + xe^{x+2y}$. [Option ID = 8644]
3. $\phi_1(y-x) + \phi_2(y+x) + e^{x+2y}$. [Option ID = 8641]
4. $\phi_1(y+x) + x\phi_2(y+x) + e^{x+2y}$. [Option ID = 8642]

Correct Answer :-

- $\phi_1(y+x) + x\phi_2(y+x) + e^{x+2y}$. [Option ID = 8642]

14) If $S = \{(1, 0, i), (1, 2, 1)\} \subseteq \mathbb{C}^3$ then $S^\perp$ is

[Question ID = 2127]

1. span $\{(i, -\frac{1}{2}(i+1), -1)\}$. [Option ID = 8506]
2. span $\{(-i, \frac{1}{2}(i+1), 1)\}$. [Option ID = 8505]
3. span $\{(i, -\frac{1}{2}(i+1), 1)\}$. [Option ID = 8507]
4. span $\{(i, \frac{1}{2}(i+1), -1)\}$. [Option ID = 8508]

Correct Answer :-

- span $\{(i, -\frac{1}{2}(i+1), 1)\}$. [Option ID = 8507]

15) The improper integral $\int_{-\infty}^{0} 2^x dx$ is

[Question ID = 2135]

1. convergent and converges to 2. [Option ID = 8540]
2. divergent. [Option ID = 8539]
3. convergent and converges to $\frac{1}{\ln 2}$. [Option ID = 8538]
4. convergent and converges to $-\ln 2$. [Option ID = 8537]

Correct Answer :-

- convergent and converges to $\frac{1}{\ln 2}$. [Option ID = 8538]

16)

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function which takes irrational values at rational points and rational values at irrational points. Then which one of the following statements is true?

[Question ID = 2145]

1. f is uniformly continuous on $\mathbb{Q}$. [Option ID = 8578]
2. f is uniformly continuous on $\mathbb{R}$. [Option ID = 8577]

3. f is uniformly continuous on $\mathbb{Q}^c$. [Option ID = 8579]
4. No such function exists. [Option ID = 8580]

Correct Answer :-

- No such function exists. [Option ID = 8580]

17) If $f: [0,10] \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} 0, & 0 \le x < 2, \\ 1, & 2 \le x \le 5 \\ 0, & 5 < x \le 10, \end{cases}$$

and $F(x) = \int_0^x f(t)dt$ then

[Question ID = 2134]

1. $F(x) = 3$ for $x \le 5$. [Option ID = 8536]
2. $F'(x) = f(x)$ for every x. [Option ID = 8534]
3. F is not differentiable at $x = 2$ and $x = 5$. [Option ID = 8535]
4. F is differentiable everywhere on $[0, 10]$. [Option ID = 8533]

Correct Answer :-

- F is not differentiable at $x = 2$ and $x = 5$. [Option ID = 8535]

18) The Maclaurin series expansion

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

is valid

[Question ID = 2136]

1. only if $x \in [-1,1]$. [Option ID = 8543]
2. if $x > -1$. [Option ID = 8541]
3. only if $x \in (-1,1]$. [Option ID = 8542]
4. for every $x \in \mathbb{R}$. [Option ID = 8544]

Correct Answer :-

- only if $x \in (-1,1]$. [Option ID = 8542]

19) If $4x \equiv 2 \pmod 6$ and $3x \equiv 5 \pmod 8$ then one of the value of x is

[Question ID = 2115]

1. 32 [Option ID = 8460]
2. 34 [Option ID = 8457]
3. 26 [Option ID = 8459]

4. 23 [Option ID = 8458]

Correct Answer :-
- 23 [Option ID = 8458]

20)

If $f(x) = \lim_{n\to\infty} S_n(x)$, where

$$S_n(x) = \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots + \frac{x}{(nx+1)((n+1)x+1)}$$

then the function f is

[Question ID = 2131]

1. continuous nowhere. [Option ID = 8524]
2. continuous everywhere. [Option ID = 8521]
3. continuous everywhere except at countably many points. [Option ID = 8522]
4. continuous everywhere except at one point. [Option ID = 8523]

Correct Answer :-
- continuous everywhere except at one point. [Option ID = 8523]

21)

The rate of change of $f(x, y) = 4y - x^2$ at the point $(1, 5)$ in the direction from $(1, 5)$ to the point $(4, 3)$ is

[Question ID = 2130]

1. $\frac{-6}{\sqrt{5}}$. [Option ID = 8519]
2. $\frac{-14}{\sqrt{13}}$. [Option ID = 8518]
3. $\frac{-12}{\sqrt{5}}$. [Option ID = 8520]
4. $\frac{-19}{\sqrt{13}}$. [Option ID = 8517]

Correct Answer :-
- $\frac{-14}{\sqrt{13}}$. [Option ID = 8518]

22) Let $G = \{a_1, a_2, \ldots, a_{25}\}$ be a group of order 25. For $b, c \in G$ let

$$bG = \{ba_1, ba_2, \ldots, ba_{25}\},\ \ Gc = \{a_1c, a_2c, \ldots, a_{25}c\}.$$

Then

[Question ID = 2119]

1. $bG = Gc$ only if $b = c$. [Option ID = 8475]
2. $bG = Gc\ \forall b, c \in G$. [Option ID = 8473]
3. $bG = Gc$ only if $b^{-1} = c$. [Option ID = 8476]
4. $bG \neq Gc$, if $b \neq c$. [Option ID = 8474]

Correct Answer :-

- $bG = Gc \ \forall b, c \in G.$ [Option ID = 8473]

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23)
If $\langle x_n \rangle$ is a sequence such that $x_n \geq 0$, for every $n \in \mathbb{N}$ and if $\lim_{n\to\infty}((-1)^n x_n)$ exists then which one of the following statements is true?

[Question ID = 2141]

1. The sequence $\langle x_n \rangle$ is a Cauchy sequence. [Option ID = 8562]
2. The sequence $\langle x_n \rangle$ is not a Cauchy sequence. [Option ID = 8564]
3. The sequence $\langle x_n \rangle$ is unbounded. [Option ID = 8563]
4. The sequence $\langle x_n \rangle$ is divergent. [Option ID = 8561]

Correct Answer :-
- The sequence $\langle x_n \rangle$ is a Cauchy sequence. [Option ID = 8562]

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24) If $n > 2$, then $n^5 - 5n^3 + 4n$ is divisible by

[Question ID = 2113]

1. 80 [Option ID = 8449]
2. 120 [Option ID = 8451]
3. 100 [Option ID = 8450]
4. 125 [Option ID = 8452]

Correct Answer :-
- 120 [Option ID = 8451]

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25) Let

$$S = \cap_{n=1}^{\infty} \left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right].$$

Then S equals

[Question ID = 2140]

1. $(2, 3]$. [Option ID = 8558]
2. $[2, 3]$. [Option ID = 8560]
3. $[2, 3)$. [Option ID = 8557]
4. $(2, 3)$. [Option ID = 8559]

Correct Answer :-
- $[2, 3]$. [Option ID = 8560]

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26) If $a_n = n^{\sin(\frac{n\pi}{2})}$ then

[Question ID = 2137]

1. $\limsup a_n = +\infty, \liminf a_n = -1.$ [Option ID = 8547]

2. $\lim\sup a_n = +\infty, \lim\inf a_n = 0.$ [Option ID = 8548]
3. $\lim\sup a_n = +\infty, \lim\inf a_n = -\infty.$ [Option ID = 8546]
4. $\lim\sup a_n = 1, \lim\inf a_n = -1.$ [Option ID = 8545]

Correct Answer :-

- $\lim\sup a_n = +\infty, \lim\inf a_n = 0.$ [Option ID = 8548]

27) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x, y) = |x| + |y|$. Then which one of the following statements is true?

[Question ID = 2129]

1. f is continuous at (0, 0) and $f_x(0,0) \neq f_y(0,0)$. [Option ID = 8515]
2. f is continuous at (0, 0) and $f_x(0,0) = f_y(0,0)$. [Option ID = 8514]
3. f is discontinuous at (0, 0) and $f_x(0,0) = f_y(0,0)$. [Option ID = 8516]
4. f is continuous at (0, 0) but f_x and f_y does not exist at (0, 0). [Option ID = 8513]

Correct Answer :-

- f is continuous at (0, 0) but f_x and f_y does not exist at (0, 0). [Option ID = 8513]

28) Let A and B be two subsets of a metric space X. If int$A$ denotes the interior A of then which one of the following statements is not true?

[Question ID = 2146]

1. $A \subseteq B \Rightarrow \text{int}A \subseteq \text{int}B$. [Option ID = 8584]
2. $\text{int}(A \cup B) = \text{int}A \cup \text{int}B$. [Option ID = 8581]
3. $\text{int}(A \cap B) = \text{int}A \cap \text{int}B$. [Option ID = 8583]
4. $\text{int}(A \cup B) \supseteq \text{int}A \cup \text{int}B$. [Option ID = 8582]

Correct Answer :-

- $\text{int}(A \cup B) = \text{int}A \cup \text{int}B$. [Option ID = 8581]

29) Which one of the following statements is false?

[Question ID = 2123]

1. A subring of a field is a subfield. [Option ID = 8490]
2. A subring of the ring of integers $\mathbb{Z}$, is an ideal of $\mathbb{Z}$. [Option ID = 8489]
3. A commutative ring with unity is a field if it has no proper ideals. [Option ID = 8492]
4. A field has no proper ideals. [Option ID = 8491]

Correct Answer :-

- A subring of a field is a subfield. [Option ID = 8490]

30) Let $\sigma = (37125)(43216) \in S_7$, the symmetric group of degree 7. The order of σ is

[Question ID = 2120]

1. 7 [Option ID = 8480]
2. 4 [Option ID = 8478]
3. 5 [Option ID = 8479]
4. 2 [Option ID = 8477]

Correct Answer :-

- 4 [Option ID = 8478]

31) Let

$$S = \cap_{n=1}^{\infty}\left[0, \frac{1}{n}\right].$$

Then which one of the following statements is true?

[Question ID = 2143]

1. $\inf S > 0.$ [Option ID = 8571]
2. $\sup S = 1$ and $\inf S = 0.$ [Option ID = 8572]
3. $\sup S > 0.$ [Option ID = 8569]
4. $\sup S = \inf S = 0.$ [Option ID = 8570]

Correct Answer :-

- $\sup S = \inf S = 0.$ [Option ID = 8570]

32) The characteristics of the partial differential equation

$$36\frac{\partial^2 z}{\partial x^2} - y^{14}\frac{\partial^2 z}{\partial y^2} - 8x^{12}\frac{\partial z}{\partial x} = 0$$

when it is of hyperbolic type are given by

[Question ID = 2160]

1. $x + \frac{36}{y^6} = c_1, x - \frac{36}{y^6} = c_2.$ [Option ID = 8638]
2. $x + \frac{1}{y^6} = c_1, x - \frac{1}{y^6} = c_2.$ [Option ID = 8637]
3. $x + \frac{1}{y^7} = c_1, x - \frac{1}{y^7} = c_2.$ [Option ID = 8639]
4. $x + \frac{36}{y^7} = c_1, x - \frac{36}{y^7} = c_2.$ [Option ID = 8640]

Correct Answer :-

- $x + \frac{1}{y^6} = c_1, x - \frac{1}{y^6} = c_2.$ [Option ID = 8637]

33) A bound for the error for the trapezoidal rule for the definite integral $\int_0^1 \frac{1}{1+x}\,dx$ is

[Question ID = 2150]

1. $\frac{1}{6}$. [Option ID = 8600]
2. $\frac{2}{25}$. [Option ID = 8597]
3. $\frac{1}{15}$. [Option ID = 8598]
4. $\frac{1}{20}$. [Option ID = 8599]

Correct Answer :-

- $\frac{1}{6}$. [Option ID = 8600]

34) Exact value of the definite integral $\int_a^b f(x)dx$ using Simpson's rule

[Question ID = 2152]

1. cannot be given for any polynomial. [Option ID = 8608]
2. is given when $f(x)$ is a polynomial of degree 4. [Option ID = 8605]
3. is given when $f(x)$ is a polynomial of degree 5. [Option ID = 8607]
4. is given when $f(x)$ is a polynomial of degree 3. [Option ID = 8606]

Correct Answer :-

- is given when $f(x)$ is a polynomial of degree 3. [Option ID = 8606]

35) Let p be a prime and let G be a non-abelian p-group. The least value of m such that $p^m \backslash o\left(\frac{G}{Z(G)}\right)$ is

[Question ID = 2121]

1. 0 [Option ID = 8481]
2. 1 [Option ID = 8482]
3. 3 [Option ID = 8484]
4. 2 [Option ID = 8483]

Correct Answer :-

- 0 [Option ID = 8481]

36) If φ is Euler's Phi function then the value of $\varphi(720)$ is

[Question ID = 2114]

1. 248 [Option ID = 8456]
2. 144 [Option ID = 8453]
3. 192 [Option ID = 8454]
4. 72 [Option ID = 8455]

Correct Answer :-

- 192 [Option ID = 8454]

37)
The total number of arithmetic operations required to find the solution of a system of n linear equations in n unknowns by Gauss elimination method is

[Question ID = 2153]

1. $\frac{2}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n.$ [Option ID = 8609]
2. $n^3 - \frac{1}{6}n.$ [Option ID = 8610]
3. $\frac{2}{3}n^3 + \frac{3}{2}n^2 - \frac{7}{6}n.$ [Option ID = 8611]
4. $\frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n.$ [Option ID = 8612]

Correct Answer :-

- $\frac{2}{3}n^3 + \frac{3}{2}n^2 - \frac{7}{6}n.$ [Option ID = 8611]

38) If $\langle x_n \rangle$ is a sequence defined as

$$x_n = \left[\frac{5+n}{2n}\right], \text{ for every } n \in \mathbb{N}$$

where [.] denotes the greatest integer function then $\lim_{n\to\infty} x_n$

[Question ID = 2142]

1. $1.$ [Option ID = 8568]
2. $\frac{1}{2}.$ [Option ID = 8566]
3. does not exist. [Option ID = 8565]
4. $0.$ [Option ID = 8567]

Correct Answer :-

- $0.$ [Option ID = 8567]

39)
Let R be a ring with characteristic n where $n \geq 2$. If M is the ring of 2×2 matrices over R then the characteristic of M is

[Question ID = 2125]

1. $1.$ [Option ID = 8500]
2. $0.$ [Option ID = 8498]
3. $n - 1.$ [Option ID = 8499]
4. $n.$ [Option ID = 8497]

Correct Answer :-

- $n.$ [Option ID = 8497]

40) If $A = \begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix}$ is a matrix with eigen values $\sqrt{6}$ and $-\sqrt{6}$, then the values of a and b are respectively,

[Question ID = 2116]

1. 2 and –1. [Option ID = 8463]
2. 2 and –2. [Option ID = 8464]
3. 2 and 1. [Option ID = 8461]
4. –2 and 1. [Option ID = 8462]

Correct Answer :-

- 2 and –2. [Option ID = 8464]

41) The dimension of the vector space of all 6 × 6 real skew-symmetric matrices is

[Question ID = 2126]

1. 36 [Option ID = 8504]
2. 21 [Option ID = 8502]
3. 30 [Option ID = 8503]
4. 15 [Option ID = 8501]

Correct Answer :-

- 15 [Option ID = 8501]

42) Let $(x_0, f(x_0)) = (0, -1), (x_1, f(x_1)) = (1, a)$ and $(x_2, f(x_2)) = (2, b)$. If the first order divided differences $f[x_0, x_1] = 5$ and $f[x_1, x_2] = c$ and the second order divided difference $f[x_0, x_1, x_2] = -\frac{3}{2}$, then the values of a, b and c are

[Question ID = 2148]

1. 4, 2, 4. [Option ID = 8592]
2. 2, 4, 6. [Option ID = 8590]
3. 4, 6, 2. [Option ID = 8589]
4. 6, 2, 4. [Option ID = 8591]

Correct Answer :-

- 4, 6, 2. [Option ID = 8589]

43) Let the polynomial $f(x) = 3x^5 + 15x^4 - 20x^3 + 10x + 20 \in \mathbb{Z}[x]$, and $f_0(x)$ be the polynomial in $\mathbb{Z}_3[x]$ obtained by reducing the coefficients of $f(x)$ modulo 3. Which one of the following statements is true?

[Question ID = 2124]

1. $f(x)$ is reducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$. [Option ID = 8496]
2. $f(x)$ is irreducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$. [Option ID = 8495]
3. $f(x)$ is reducible over $\mathbb{Q}$, $f_0(x)$ is irreducible over $\mathbb{Z}_3$. [Option ID = 8494]
4. $f(x)$ is irreducible over $\mathbb{Q}$, $f_0(x)$ is irreducible over $\mathbb{Z}_3$. [Option ID = 8493]

Correct Answer :-

- $f(x)$ is irreducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$. [Option ID = 8495]

44) The general solution of the system of the differential equations

$$x_1' = 3x_1 - 2x_2$$
$$x_2' = 2x_1 - 2x_2$$

is given by

[Question ID = 2158]

1. $\begin{pmatrix} c_1e^{-t} + 2c_2e^{2t} \\ 2c_1e^{-t} + c_2e^{2t} \end{pmatrix}$. [Option ID = 8632]
2. $\begin{pmatrix} c_1e^{t} + 2c_2e^{-2t} \\ 2c_1e^{t} + 2c_2e^{-2t} \end{pmatrix}$. [Option ID = 8631]
3. $\begin{pmatrix} c_1e^{t} + 2c_2e^{-2t} \\ c_1e^{t} + c_2e^{-2t} \end{pmatrix}$. [Option ID = 8629]
4. $\begin{pmatrix} c_1e^{-t} + c_2e^{2t} \\ c_1e^{-t} - c_2e^{2t} \end{pmatrix}$. [Option ID = 8630]

Correct Answer :-

- $\begin{pmatrix} c_1e^{-t} + 2c_2e^{2t} \\ 2c_1e^{-t} + c_2e^{2t} \end{pmatrix}$. [Option ID = 8632]

45) The eigenvalues for the Sturm–Liouville problem

$$y'' + \lambda y = 0, 0 \le x \le \pi,$$
$$y(0) = 0, y'(\pi) = 0$$

are [Question ID = 2155]

1. $\lambda_n = n^2\pi^2, n = 1, 2, \dots..$ [Option ID = 8619]
2. $\lambda_n = n^2, n = 1, 2, \dots..$ [Option ID = 8618]
3. $\lambda_n = n\pi, n = 1, 2, \dots..$ [Option ID = 8617]
4. $\lambda_n = \frac{(2n-1)^2}{4}, n = 1, 2, \dots..$ [Option ID = 8620]

Correct Answer :-

- $\lambda_n = \frac{(2n-1)^2}{4}, n = 1, 2, \dots..$ [Option ID = 8620]

46)

The initial value problem

$$x\frac{dy}{dx} - 2y = 0,$$
$$x > 0, y(0) = 0$$

has

[Question ID = 2157]

1. exactly two solutions [Option ID = 8626]
2. a unique solution. [Option ID = 8627]
3. no solution. [Option ID = 8628]
4. infinitely many solutions. [Option ID = 8625]

Correct Answer :-

- infinitely many solutions. [Option ID = 8625]

47) The partial differential equation

$$(x^2 - 1)\frac{\partial^2 z}{\partial x^2} + 2y\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = 0$$

is

[Question ID = 2162]

1. hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. [Option ID = 8645]
2. parabolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. [Option ID = 8646]
3. hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$. [Option ID = 8648]
4. elliptic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$. [Option ID = 8647]

Correct Answer :-

- hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$. [Option ID = 8648]

48)

Let f be a convex function with $f(0) = 0$. Then the function g defined on $(0, +\infty)$ as $g(x) = \frac{f(x)}{x}$

[Question ID = 2132]

1. is an increasing function. [Option ID = 8525]
2. is such that its monotonicity cannot be determined. [Option ID = 8528]
3. is neither increasing nor decreasing function. [Option ID = 8527]
4. is a decreasing function. [Option ID = 8526]

Correct Answer :-

- is an increasing function. [Option ID = 8525]

49) Which one of the statements is false? [Question ID = 2117]

1. Every quotient group of a cyclic group is cyclic. [Option ID = 8465]
2. If G and H are groups and $f : G \to H$ is a homomorphism then f induces an isomorphism of $\frac{G}{\mathrm{Ker}(f)}$ with H. [Option ID = 8467]
3. Every quotient group of an abelian group is abelian. [Option ID = 8468]

4. If G is a group and $Z(G)$ is its centre such that the quotient group of G by $Z(G)$ is cyclic, then G is abelian. [Option ID = 8466]

Correct Answer :-

- If G and H are groups and $f : G \to H$ is a homomorphism then f induces an isomorphism of $\frac{G}{\mathrm{Ker}(f)}$ with H. [Option ID = 8467]

50) For cubic spline interpolation which one of the following statements is true? [Question ID = 2149]

1. The second derivatives of the splines are continuous at the interior data points but not the first derivatives. [Option ID = 8594]
2. The third derivatives of the splines are continuous at the interior data points. [Option ID = 8596]
3. The first derivatives of the splines are continuous at the interior data points but not the second derivatives. [Option ID = 8593]
4. The first and the second derivatives of the splines are continuous at the interior data points. [Option ID = 8595]

Correct Answer :-

- The first and the second derivatives of the splines are continuous at the interior data points. [Option ID = 8595]