



DU MPhil Phd in Mathematics

Topic:- DU\_J18\_MPHIL\_MATHS\_Topic01

1) The mathematician who was awarded Abel's prize for a proof of Fermat's Last Theorem is [Question ID = 19249]

1. Andrew Wiles. [Option ID = 46987]
2. Johan F. Nash. [Option ID = 46988]
3. S. R. Srinivasa Varadhan. [Option ID = 46989]
4. Lennart Carleson. [Option ID = 46990]

Correct Answer :-

- Andrew Wiles. [Option ID = 46987]

2) Founder of Indian Mathematical Society(IMS) was [Question ID = 19252]

1. Asutosh Mukherjee. [Option ID = 47000]
2. S. Narayana Aiyer. [Option ID = 47001]
3. M.T. Narayaniyengar. [Option ID = 47002]
4. V. Ramaswamy Aiyer. [Option ID = 46999]

Correct Answer :-

- V. Ramaswamy Aiyer. [Option ID = 46999]

3) Let R be a commutative ring with identity. If R is an Artinian domain, then the total number of prime ideals in R is [Question ID = 19280]

1. 1 [Option ID = 47111]
2. infinite. [Option ID = 47114]
3. 3 [Option ID = 47113]
4. 2 [Option ID = 47112]

Correct Answer :-

- 1 [Option ID = 47111]

4) Riemann hypothesis is associated with the function [Question ID = 19250]

1.  $f(s) = \int_0^\infty t^{s-1} e^{-t} dt.$  [Option ID = 46991]
2.  $f(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt$  [Option ID = 46992]
3. Hermite polynomial [Option ID = 46994]
4.  $f(s) = \sum_{n=1}^\infty \frac{1}{n^s}, s \in \mathbb{C}$  [Option ID = 46993]

Correct Answer :-

- $f(s) = \sum_{n=1}^\infty \frac{1}{n^s}, s \in \mathbb{C}$  [Option ID = 46993]

5) For the stream function of a two dimensional motion, which of the following is not true

[Question ID = 19297]

1. Stream function is constant along a stream line. [Option ID = 47181]
2. Stream function is harmonic. [Option ID = 47180]
3. Stream function exists for steady motion of compressible fluid. [Option ID = 47179]
4. Stream function has dimension  $L^2 T^{-2}$ . [Option ID = 47182]

Correct Answer :-

- Stream function has dimension  $L^2 T^{-2}$ . [Option ID = 47182]

6) The famous Indian mathematician Srinivas Ramanujan passed away in the year [Question ID = 19248]

1. 1920 [Option ID = 46984]
2. 1922 [Option ID = 46985]
3. 1921 [Option ID = 46983]
4. 1919 [Option ID = 46986]

Correct Answer :-

- 1920 [Option ID = 46984]

7) Let F be a finite field with 9 elements. How many elements of F have order 9? [Question ID = 19287]



1. 1 [Option ID = 47142]
2. 4 [Option ID = 47140]
3. 8 [Option ID = 47139]
4. 2 [Option ID = 47141]

Correct Answer :-

- 4 [Option ID = 47140]

8) For a viscous compressible fluid Consider the following statements:

- (I) Stress matrix is symmetric.
- (II) Kinematic coefficient of viscosity is dependent on the mass.
- (III) Rate of dilatation is  $\nabla \cdot \bar{q}$ .

Then

[Question ID = 19293]

1. all of I, II and III are true. [Option ID = 47163]
2. only I and III are true. [Option ID = 47164]
3. only I and II are true. [Option ID = 47165]
4. only II and III are true. [Option ID = 47166]

Correct Answer :-

- only I and III are true. [Option ID = 47164]

9) Let  $f : R \rightarrow R'$  be a ring homomorphism. Assume that 1 and 1' are multiplicative identities of the rings  $R$  and  $R'$  respectively. Then  $f(1) = 1'$  if

- I  $f$  is onto.
- II  $f$  is one-one.
- III  $R$  is a domain.
- IV  $R'$  is a domain.

The correct options are

[Question ID = 19276]

1. III and IV only. [Option ID = 47096]
2. II and III only [Option ID = 47098]
3. I and IV only. [Option ID = 47097]
4. I and II only. [Option ID = 47095]

Correct Answer :-

- I and IV only. [Option ID = 47097]

10) For a solid stationary sphere of radius  $a$  placed in an incompressible fluid of uniform stream with velocity  $-Ui$ :

- (I) velocity potential  $\phi(r, \theta) = U \cos \theta (r + \frac{a^3}{2r^2})$ .
- (II) there exist two stagnation points  $(a, 0), (a, \pi)$ .
- (III) stagnation pressure  $p_\infty + \frac{1}{2}\rho U^2$ ,  $p_\infty$  is a pressure at  $\infty$ .
- (IV) velocity at any point of surface of sphere is  $(0, U \sin \theta, 0)$ .

Then

[Question ID = 19296]

1. only I, II, IV are true. [Option ID = 47175]
2. only I, III, IV are true. [Option ID = 47177]
3. only I, II, III are true. [Option ID = 47176]
4. only II, III, IV are true. [Option ID = 47178]

Correct Answer :-

- only I, II, III are true. [Option ID = 47176]

11) Let  $R = \{a + ib : a, b \in \mathbb{Z}\}$ . Then  $R$  is a Euclidean domain with

[Question ID = 19277]

1. exactly two units. [Option ID = 47099]
2. exactly eight units. [Option ID = 47101]
3. exactly four units. [Option ID = 47100]
4. infinitely many units. [Option ID = 47102]

Correct Answer :-

- exactly four units. [Option ID = 47100]

12) Consider the sequence of Lebesgue measurable functions  $(f_n)$  on  $\mathbb{R}$

$$f_n(x) = \begin{cases} 5, & x \geq 2^n \\ 0, & x < 2^n. \end{cases}$$

Then  $\lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} f_n(x) dx$

[Question ID = 19263]

1. does not exist [Option ID = 47046]
2. equals 0. [Option ID = 47043]
3. equals 5. [Option ID = 47044]
4. equals  $\infty$ . [Option ID = 47045]

Correct Answer :-

- equals  $\infty$ . [Option ID = 47045]

13) Let  $f(x) = \sin x + \cos x$  on  $[0, \pi]$ . Then  $\|f\|_{\infty}$  is equal to

[Question ID = 19269]

1. 1 [Option ID = 47067]
2.  $2\sqrt{2}$  [Option ID = 47070]
3.  $\sqrt{2}$  [Option ID = 47068]
4.  $1/\sqrt{2}$  [Option ID = 47069]

Correct Answer :-

- $\sqrt{2}$  [Option ID = 47068]

14) Let  $f$  be a continuous function on a finite interval  $[a, b]$ . Then

$$\lim_{t \rightarrow \infty} \int_a^b f(x) \sin tx dx$$

[Question ID = 19260]

1. equals 0 [Option ID = 47033]
2. equals  $\sup_{x \in [a, b]} f(x)$  [Option ID = 47034]
3. does not exist [Option ID = 47032]
4. equals  $\int_a^b f(x) dx$ . [Option ID = 47031]

Correct Answer :-

- equals 0 [Option ID = 47033]

15)

Let  $(X, d)$  be a metric space and  $A \subseteq X, B \subseteq X$ . Consider the following statements:

I If  $x \notin A$  then  $d(x, A) > 0$ .

II If  $A \cap B = \phi$ , then  $d(A, B) \geq 0$ .

III If  $A$  is closed and  $x \notin A$  then  $d(x, A) > 0$ .

IV If  $A$  and  $B$  are closed and  $A \cap B = \phi$  then  $d(A, B) \geq 0$ .

Then,

[Question ID = 19259]

1. all statements are correct. [Option ID = 47030]
2. only III is correct. [Option ID = 47028]
3. only II, III, IV are correct. [Option ID = 47027]
4. only III and IV are correct. [Option ID = 47029]

Correct Answer :-

16) The set  $A = \{x \in \mathbb{Q} \mid -\sqrt{7} \leq x \leq \sqrt{7}\}$  in the subspace  $\mathbb{Q}$  of the real line  $\mathbb{R}$  is

[Question ID = 19271]

1. neither open nor closed [Option ID = 47078]
2. open but not closed [Option ID = 47075]
3. both open and closed [Option ID = 47077]
4. closed but not open [Option ID = 47076]

Correct Answer :-

- both open and closed [Option ID = 47077]

17) A Lipschitz's constant associated with the function  $f(x, y) = y^{2/3}$  on  $R : |x| \leq 1, |y| \leq 1$

[Question ID = 19288]

1. does not exist. [Option ID = 47146]
2. equals  $1/2$ . [Option ID = 47145]
3. equals 0. [Option ID = 47143]
4. equals 1. [Option ID = 47144]

Correct Answer :-

- does not exist. [Option ID = 47146]

18) Let  $I = \int_C y dx + (x + 2y) dy$ , where  $C = C_1 + C_2$ ,  $C_1$  being the line joining  $(0, 1)$  to  $(1, 1)$  and  $C_2$  is the line joining  $(1, 1)$  to  $(1, 0)$ . The value of  $I$  is

[Question ID = 19256]

1. 2 [Option ID = 47017]
2. -1 [Option ID = 47018]
3. 1 [Option ID = 47015]
4. 0 [Option ID = 47016]

Correct Answer :-

- 1 [Option ID = 47015]

19) Let  $F(x) = \int_0^x \frac{\sin t}{t^{3/2}} dt$ ,  $0 < x < \infty$ . The local maximum value is at the point

[Question ID = 19255]

1.  $x = \pi/2$  [Option ID = 47013]
2.  $x = 4\pi$  [Option ID = 47014]

3.  $x = \pi$  [Option ID = 47011]

4.  $x = 2\pi$  [Option ID = 47012]

Correct Answer :-

1.  $x = \pi$  [Option ID = 47011]

20) The general integral of the partial differential equation  $yzp + xzq = xy$ , where  $p = \frac{\partial z}{\partial x}$ ,  $q = \frac{\partial z}{\partial y}$  ( $G$  being an arbitrary function) is

[Question ID = 19289]

1.  $z^2 = x^2 - G(x^2 + y^2)$ . [Option ID = 47150]

2.  $z^2 = y^2 + G(x^2 + y^2)$ . [Option ID = 47147]

3.  $z^2 = y^2 + G(x^2 - y^2)$ . [Option ID = 47149]

4.  $z^2 = x - G(x^2 - y^2)$ . [Option ID = 47148]

Correct Answer :-

2.  $z^2 = y^2 + G(x^2 - y^2)$ . [Option ID = 47149]

21) Let  $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ . Then

[Question ID = 19257]

1. For any  $\delta > 0$ ,  $f$  is not monotonic on  $[0, \delta)$  [Option ID = 47020]

2.  $f$  has a local extremum at  $x = 0$  [Option ID = 47021]

3. For any  $\delta > 0$ ,  $f$  is convex on  $[0, \delta)$  [Option ID = 47022]

4.  $f'$  is continuous at  $x = 0$  [Option ID = 47019]

Correct Answer :-

1. For any  $\delta > 0$ ,  $f$  is not monotonic on  $[0, \delta)$  [Option ID = 47020]

22) Let  $F = \mathbb{Q}((\sqrt{2}, \sqrt{3}))$ . Then  $F$  is minimal splitting field of the polynomial  $(x^2 - 2)(x^2 - 3)$  over  $\mathbb{Q}$ . The field  $F$  is not the minimal splitting field of which of the following polynomials over  $\mathbb{Q}$

[Question ID = 19286]

1.  $x^4 - 10x^2 + 1$ . [Option ID = 47135]

2.  $x^{-4} - x^2 + 6$ . [Option ID = 47137]

3.  $x^4 + x^2 + 1$ . [Option ID = 47136]

4.  $x^4 + x^2 + 25$ . [Option ID = 47138]

Correct Answer :-

An elementary solution of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is of the form ( $\bar{r} = xi + yj$ ,  $\bar{r}' = x'i + y'j$ )

[Question ID = 19290]

1.  $u = \log |\bar{r}\bar{r}'|$ . [Option ID = 47154]
2.  $u = \log \frac{1}{|\bar{r}+\bar{r}'|}$ . [Option ID = 47151]
3.  $u = \log \frac{1}{|\bar{r}\bar{r}'|}$ . [Option ID = 47153]
4.  $u = \log \frac{1}{|\bar{r}-\bar{r}'|}$ . [Option ID = 47152]

Correct Answer :-

- $u = \log \frac{1}{|\bar{r}-\bar{r}'|}$ . [Option ID = 47152]

24)  
Let  $E = \{x \in (0, \sqrt{2}] : x \text{ is a rational number}\} \cup \{y \in [2, 3] : y \text{ is an irrational number}\}$   
Then the Lebesgue measure of  $E$  is

[Question ID = 19264]

1. 1 [Option ID = 47048]
2.  $\sqrt{2}$  [Option ID = 47049]
3.  $1/2$  [Option ID = 47050]
4.  $\sqrt{2} + 1$  [Option ID = 47047]

Correct Answer :-

- 1 [Option ID = 47048]

25)  
Let  $H$  be a Sylow  $p$ -subgroup and  $K$  be a  $p$ -subgroup of a finite group  $G$ . Which of the following is incorrect is incorrect ( $H \text{ char } G$  means  $H$  is characteristic in  $G$ )

[Question ID = 19282]

1.  $K \triangleleft G \Rightarrow K \subset H$ . [Option ID = 47119]
2.  $K \triangleleft G \Rightarrow K \text{ char } H$ . [Option ID = 47121]
3.  $K \subset H$  if  $K \triangleleft G$ . [Option ID = 47120]
4.  $K \triangleleft G \not\Rightarrow H \cap K \triangleleft H$ . [Option ID = 47122]

Correct Answer :-

- $K \triangleleft G \not\Rightarrow H \cap K \triangleleft H$ . [Option ID = 47122]

26)



A two dimensional motion with complex potential  $w = U(z + \frac{a^2}{z}) + ik \log \frac{z}{a}$  has

- (I) stream lines as circle  $|z| = a$ .
- (II) circulation zero about circle  $|z| = a$ .
- (III) has two stagnation points in general.
- (IV) velocity at infinity equal to  $(-U)$ .

Then

[Question ID = 19295]

- 1. only I, II, IV are true. [Option ID = 47172]
- 2. only I, III, IV are true. [Option ID = 47173]
- 3. only I, II, III are true. [Option ID = 47171]
- 4. only II, III, IV are true. [Option ID = 47174]

Correct Answer :-

- only I, III, IV are true. [Option ID = 47173]

27)

Let  $G$  be an abelian group of order 15. Define a map  $\phi : G \rightarrow G$  by  $\phi(g) = g^8$  for all  $g \in G$ . Consider the statements:

- I  $\phi$  is a homomorphism.
- II  $\phi$  is one-to-one.
- III  $\phi$  is onto.

Then

[Question ID = 19281]

- 1. only I and III are true. [Option ID = 47117]
- 2. only I and II are true. [Option ID = 47116]
- 3. only I is true. [Option ID = 47115]
- 4. all statements are true. [Option ID = 47118]

Correct Answer :-

- all statements are true. [Option ID = 47118]

28)

Let  $\xi$  be a primitive  $n^{\text{th}}$  root of unity where  $n \equiv 2 \pmod{4}$ . Then  $[\mathbb{Q}(\xi) : \mathbb{Q}(\xi^2)]$  is

(Here  $[V : F]$  denotes the dimension of the vector space  $V$  over  $F$ )

[Question ID = 19285]

- 1. 1 [Option ID = 47131]
- 2. 2 [Option ID = 47132]
- 3.  $\phi(n)$  [Option ID = 47133]
- 4.  $\phi(n)/2$  [Option ID = 47134]

Correct Answer :-

- 1 [Option ID = 47131]

29)

The closed topologist's sine curve  $\{(x, \sin \frac{1}{x}) \mid x \in (0, 1]\}$  as subspace of real line  $\mathbb{R}$  is

[Question ID = 19272]

- 1. a path connected space [Option ID = 47081]
- 2. connected but not locally connected [Option ID = 47079]

3. a locally path connected space [Option ID = 47082]  
 4. locally connected but not connected [Option ID = 47080]

Correct Answer :-

- connected but not locally connected [Option ID = 47079]

30)

Let  $R(T)$  and  $N(T)$  denote the range space and null space of the linear transformation  $T : P_2(\mathbb{R}) \rightarrow M_{2 \times 2}(\mathbb{R})$  which is given by

$$T(f) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & f(0) \end{pmatrix}.$$

Then

[Question ID = 19275]

1.  $\dim(R(T)) = 2$  and  $\dim(N(T)) = 1$  [Option ID = 47094]
2.  $\dim(R(T)) = 0$  and  $\dim(N(T)) = 2$  [Option ID = 47093]
3.  $\dim(R(T)) = 2$  and  $\dim(N(T)) = 0$  [Option ID = 47091]
4.  $\dim(R(T)) = 1$  and  $\dim(N(T)) = 1$  [Option ID = 47092]

Correct Answer :-

- $\dim(R(T)) = 2$  and  $\dim(N(T)) = 1$  [Option ID = 47094]

- 31) The bilinear transformation on  $\mathbb{C}$  which maps  $z = 0, -i, -1$  into  $w = i, 1, 0$  is

[Question ID = 19265]

1.  $-i \frac{z+1}{z-1}$  [Option ID = 47053]
2.  $\frac{z+1}{z-1}$  [Option ID = 47052]
3.  $i \frac{z+1}{z-1}$  [Option ID = 47051]
4.  $i \frac{z-1}{z+1}$  [Option ID = 47054]

Correct Answer :-

- $-i \frac{z+1}{z-1}$  [Option ID = 47053]

- 32) Let  $A, B \in M_n(\mathbb{C})$ . Consider the following statements

I If  $A, B$  and  $A + B$  are invertible, then  $A^{-1} + B^{-1}$  is invertible.

II If  $A, B$  and  $A + B$  are invertible, then  $A^{-1} - B^{-1}$  is invertible.

III If  $AB$  is nilpotent, then  $BA$  is nilpotent.

IV Characteristic polynomials of  $AB$  and  $BA$  are equal if  $A$  is invertible.

Then

[Question ID = 19274]

1. only I, III, and IV are true [Option ID = 47089]
2. all the statements are true. [Option ID = 47090]
3. only III is true [Option ID = 47088]
4. only I and II are true [Option ID = 47087]

Correct Answer :-

- only I, III, and IV are true [Option ID = 47089]



33)

For the boundary value problem:  $L(y) = y'' = 0$ ,  $y(0) = 0$ ,  $y'(1) = 0$ , the Green's function is

[Question ID = 19291]

1.  $G(x, \xi) = \begin{cases} \xi, & x \leq \xi \\ x, & x > \xi \end{cases}$  [Option ID = 47156]

2.  $G(x, \xi) = \begin{cases} -x, & x \leq \xi \\ -\xi, & x > \xi \end{cases}$  [Option ID = 47157]

3.  $G(x, \xi) = \begin{cases} -x, & x \leq \xi \\ -\xi, & x > \xi \end{cases}$  [Option ID = 47158]

4.  $G(x, \xi) = \begin{cases} x, & x \leq \xi \\ \xi, & x > \xi \end{cases}$  [Option ID = 47155]

Correct Answer :-

•  $G(x, \xi) = \begin{cases} x, & x \leq \xi \\ \xi, & x > \xi \end{cases}$  [Option ID = 47155]

34) Let  $E = \{x \in [0, \pi) : \sin 4x < 0\}$ . Then Lebesgue measure of  $E$  is

[Question ID = 19262]

1.  $\pi/2$  [Option ID = 47040]

2.  $\pi/4$  [Option ID = 47039]

3.  $3\pi/4$  [Option ID = 47041]

4.  $\pi/3$  [Option ID = 47042]

Correct Answer :-

•  $\pi/2$  [Option ID = 47040]

35)

Let  $x_1, x_2, \dots, x_n$  be non-zero real numbers. With  $x_{ij} = x_i x_j$ , let  $X$  be the  $n \times n$  matrix  $(x_{ij})$ . Then

[Question ID = 19273]

1. the matrix  $X$  is positive definite if  $(x_1, x_2, \dots, x_n)$  is a non-zero vector [Option ID = 47084]

2. the matrix  $X$  is positive semi definite for all  $(x_1, x_2, \dots, x_n)$  [Option ID = 47085]

3. for all  $(x_1, x_2, \dots, x_n)$ , zero is an eigenvalue of  $X$ . [Option ID = 47086]

4. it is possible to chose  $x_1, x_2, \dots, x_n$  so as to make the matrix  $X$  non singular [Option ID = 47083]

Correct Answer :-

36)

Let  $A = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ is continuous on } \mathbb{Q} \text{ and discontinuous on } \mathbb{Q}'\}$ , where  $\mathbb{Q}$  is the set of all rational numbers and  $\mathbb{Q}'$  is the set of all irrational numbers. Let  $\mu$  be a counting measure on  $A$ . Then

[Question ID = 19258]

1.  $\mu(A) = \sum_{q \in \mathbb{Q}} \frac{1}{2^q}$  [Option ID = 47026]
2.  $\mu(A)$  is infinite [Option ID = 47023]
3.  $\mu(A) = 0$  [Option ID = 47024]
4.  $\mu(A) = 2$  [Option ID = 47025]

Correct Answer :-

3.  $\mu(A) = 0$  [Option ID = 47024]

37) Let  $R = \mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5$ . Then the total number of zero divisors in  $R$  is

[Question ID = 19278]

1. 15 [Option ID = 47106]
2. 10 [Option ID = 47105]
3. 20 [Option ID = 47104]
4. 22 [Option ID = 47103]

Correct Answer :-

38) Let  $a, b \in \mathbb{C}$  such that  $0 < |a| < |b|$ . Then the Laurent expression of  $\frac{1}{(z-a)(z-b)}$  in the annulus  $|a| < |z| < |b|$  is

[Question ID = 19266]

1.  $\frac{1}{a-b} \left[ \sum_{n=0}^{\infty} \frac{z^n}{b^n} + \sum_{n=0}^{\infty} \frac{a^n}{z^{n+1}} \right]$  [Option ID = 47057]
2.  $\frac{1}{a-b} \left[ \sum_{n=0}^{\infty} \frac{z^n}{b^{n+1}} + \sum_{n=0}^{\infty} \frac{a^n}{z^{n+1}} \right]$  [Option ID = 47055]
3.  $\frac{1}{a-b} \left[ \sum_{n=0}^{\infty} \frac{z^n}{a^{n+1}} + \sum_{n=0}^{\infty} \frac{b^n}{z^{n+1}} \right]$  [Option ID = 47056]
4.  $\frac{1}{a-b} \left[ \sum_{n=0}^{\infty} \frac{z^n}{a^n} + \sum_{n=0}^{\infty} \frac{b^{n+1}}{z^n} \right]$  [Option ID = 47058]

Correct Answer :-

2.  $\frac{1}{a-b} \left[ \sum_{n=0}^{\infty} \frac{z^n}{b^{n+1}} + \sum_{n=0}^{\infty} \frac{a^n}{z^{n+1}} \right]$  [Option ID = 47055]

39) Consider the following statements:

I  $x^3 - 9$  is not irreducible over  $\mathbb{Z}_7$ .

II  $x^3 - 9$  is not irreducible over  $\mathbb{Z}_{11}$ .

Then

[Question ID = 19279]

1. II is false but I is true. [Option ID = 47107]
2. both I and II are true. [Option ID = 47109]
3. both I and II are false. [Option ID = 47110]

4. I is false but II is true. [Option ID = 47108]

Correct Answer :-

• I is false but II is true. [Option ID = 47108]

40) The contour integral  $\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ , where  $C$  is the circle  $|z| = 4$  taken anti-clockwise equals

[Question ID = 19267]

1.  $\frac{i}{2\pi}$  [Option ID = 47061]
2.  $\frac{4}{\pi i}$  [Option ID = 47059]
3.  $\frac{4}{\pi}$  [Option ID = 47060]
4.  $\frac{i}{\pi}$  [Option ID = 47062]

Correct Answer :-

•  $\frac{i}{\pi}$  [Option ID = 47062]

41) The pressure  $p(x, y, z)$  in steady flow of inviscid incompressible fluid of density  $\rho$  with velocity  $\vec{q} = (kx, -ky, 0)$ ,  $k$  is a constant, under no external force when  $p(0, 0, 0) = p_0$ , is

[Question ID = 19341]

1.  $p_0 - \rho k^2(y^2 - x^2)/2$ . [Option ID = 47358]
2.  $p_0 - \rho k^2(y^2 + x^2)$ . [Option ID = 47354]
3.  $p_0 - \rho k^2(y^2 - x^2)$ . [Option ID = 47352]
4.  $p_0 - \rho k^2(y^2 + x^2)/2$ . [Option ID = 47356]

Correct Answer :-

•  $p_0 - \rho k^2(y^2 + x^2)/2$ . [Option ID = 47356]

42) let  $E$  be a Lebesgue non-measurable subset of  $\mathbb{R}$ . Define  $f : \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) = \begin{cases} 2, & x \in E \\ -2, & x \in E^c. \end{cases}$$

Then

[Question ID = 19261]

1. neither  $f$  nor  $|f|$  is Lebesgue measurable [Option ID = 47038]
2.  $f$  is Lebesgue measurable but  $|f|$  is not Lebesgue measurable [Option ID = 47036]
3.  $f$  is not Lebesgue measurable but  $|f|$  is Lebesgue measurable [Option ID = 47037]
4.  $f$  and  $|f|$  both are Lebesgue measurable. [Option ID = 47035]

Correct Answer :-

$f$  is not Lebesgue measurable but  $|f|$  is Lebesgue measurable [Option ID = 47037]

43) Every non trivial solution of the equation  $y'' + (\sinh x)y = 0$  has

[Question ID = 19292]

1. only finitely many zeros in  $(0, \infty)$ . [Option ID = 47162]
2. infinitely many zeros in  $(-\infty, 0)$ . [Option ID = 47160]
3. infinitely many zeros in  $(0, \infty)$ . [Option ID = 47159]
4. at most one zero in  $(0, \infty)$ . [Option ID = 47161]

Correct Answer :-

infinitely many zeros in  $(0, \infty)$ . [Option ID = 47159]

44) Which of the following statements is true [Question ID = 19253]

1. If  $0 \leq a_n \leq b_n$  and  $\sum b_n$  diverges then  $\sum a_n$  diverges [Option ID = 47005]
2. If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\sum \frac{a_n}{a_n^2 + n^2}$  converges [Option ID = 47004]
3.  $\sum_{k=1}^{\infty} \left( \tan^{-1} \frac{1}{k} - \tan^{-1} \frac{1}{k+1} \right) = \frac{\pi}{8}$  [Option ID = 47003]
4.  $\sum_{n=1}^{\infty} \frac{1}{n^n} \geq 2$  [Option ID = 47006]

Correct Answer :-

If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\sum \frac{a_n}{a_n^2 + n^2}$  converges [Option ID = 47004]

45) Which of the following statements is not true [Question ID = 19254]

1. The set of all algebraic numbers is countable. [Option ID = 47010]
2. The set of rational numbers is equivalent to the set of natural numbers [Option ID = 47008]
3. Given a set  $A$ , there exists a function  $f : A \rightarrow P(A)$  that is onto ( $P(A)$  denotes power set of  $A$ ) [Option ID = 47009]
4. There is one-one function taking  $(-1, 1)$  onto  $\mathbb{R}$ . [Option ID = 47007]

Correct Answer :-

Given a set  $A$ , there exists a function  $f : A \rightarrow P(A)$  that is onto ( $P(A)$  denotes power set of  $A$ ) [Option ID = 47009]

46) Which of the following statements is not true [Question ID = 19270]

1. An uncountable discrete space is not separable. [Option ID = 47072]
2. Every closed subspace of a separable space is separable. [Option ID = 47073]
3. Every compact metric space is Lindelöf. [Option ID = 47074]
4. Every second countable space is separable. [Option ID = 47071]

Correct Answer :-

Every closed subspace of a separable space is separable. [Option ID = 47073]

47) Which of the following is not correct (Here  $[V : F]$  denotes the dimension of the vector space  $V$  over  $F$ ) [Question ID = 19284]

1.  $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, i, \sqrt{6}) : \mathbb{Q}] = 16$ . [Option ID = 47130]
2.  $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, i) : \mathbb{Q}] = 8$ . [Option ID = 47129]
3.  $[\mathbb{Q}(\sqrt{3}) : \mathbb{Q}] = 2$ . [Option ID = 47127]

4.  $[\mathbb{Q}(\sqrt{3}, i) : \mathbb{Q}] = 4.$  [Option ID = 47128]

Correct Answer :-

•  $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, i, \sqrt{6}) : \mathbb{Q}] = 16.$  [Option ID = 47130]

48) Which of the following Banach spaces is not a Hilbert space [Question ID = 19258]

1.  $(L^2([0, 1]), \|\cdot\|_2)$  [Option ID = 47064]

2.  $\mathbb{R}^n$  with the norm  $\|x\| = \sqrt{\xi_1^2 + \xi_2^2 + \cdots + \xi_n^2}$ , where  $x = (\xi_1, \xi_2, \cdots, \xi_n)$  [Option ID = 47065]

3.  $\mathbb{R}^n$  with the norm  $\|x\| = \max\{|\xi_1|, |\xi_2|, \cdots, |\xi_n|\}$ , where  $x = (\xi_1, \xi_2, \cdots, \xi_n)$  [Option ID = 47066]

4.  $(l^2, \|\cdot\|_2)$  [Option ID = 47063]

Correct Answer :-

•  $\mathbb{R}^n$  with the norm  $\|x\| = \max\{|\xi_1|, |\xi_2|, \cdots, |\xi_n|\}$ , where  $x = (\xi_1, \xi_2, \cdots, \xi_n)$  [Option ID = 47066]

49) Which of the following websites is of Mathematical Reviews [Question ID = 19251]

1. <https://mathscinet.ams.org> [Option ID = 46997]

2. <https://mathscinet.ac.in> [Option ID = 46995]

3. <https://math.ac.au> [Option ID = 46996]

4. <https://www.mathjournal.org> [Option ID = 46998]

Correct Answer :-

• <https://mathscinet.ams.org> [Option ID = 46997]

50) Let  $G$  be a cyclic group of order 42. The number of distinct composition series of  $G$  is [Question ID = 19283]

1. 8 [Option ID = 47126]

2. 16 [Option ID = 47123]

3. 10 [Option ID = 47125]

4. 6 [Option ID = 47124]

Correct Answer :-

• 6 [Option ID = 47124]