B. TECH.

THEORY EXAMINATION (SEM–VIII) 2016-17

APPLIED LINEAR ALGEBRA

Time : 3 Hours *Max. Marks : 100*

Note : *Be precise in your answer. In case of numerical problem assume data wherever not provided.*

SECTION – A

1. Attempt all parts of the following questions: 10 x 2 = 20

a) Find dimension of vector space $C(R)$.

b) Define Basis of a vector space.

c) State rank-nullity theorem.

d) Let $T: R^2 \to R^2$ be a linear transformation such that $T\left[\begin{pmatrix}1\\0\end{pmatrix}\right]=\begin{pmatrix}1\\2\end{pmatrix}$ and $T\left[\begin{pmatrix}0\\1\end{pmatrix}\right]=\begin{pmatrix}2\\1\end{pmatrix}$ What is the value of $T\left[\begin{pmatrix}1\\1\end{pmatrix}\right]$.

e) Find all non-singular linear transformation $T: R^4 \to R^3$.

f) Find the condition that T is non-singular.

g) Define complete ortho normal set.

h) Give polarization identity.

i) A real quadratic form in three variables is equivalent to the diagonal form $6y_1^2 + 3y_2^2 + 0y_3^2$ then find the quadratic form.

j) Define linear functionals with examples.

SECTION – B

2. Attempt any five parts of the following questions: 5 x 10 = 50

a) Define field with example.

b) Prove that the set $V = \left\{\begin{bmatrix} a & a+b \\ a+b & b\end{bmatrix} : a,b \in R\right\}$ is vector space over R.

c) Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

d) The matrix of quadratic form q on R^3 given by $q(x_1, x_2, x_3) = x_1^2 - x_3^2 + 3x_1x_2 - 6x_2x_3$

e) State and prove Minkowski inequality.

f) Let T be the linear transformation on V such that $T^3 - T^2 - T + I = 0$, then find T^{-1}.

g) Let V be a finite dimensional inner product space and S, S_1, S_2 are subset of V Prove that (i) $S^{\perp} = \{S\}^T$ (ii) $\{S\} = S^{\perp\perp}$

h) Prove that union of two subspaces is subspace if one is contained in other.

SECTION – C

Attempt any two parts of the following questions: **2 x 15 = 30**

3. (i) Prove that the system of three vectors $(1,3,2),(1,-7,-8),\ (2,1,-1)$ of $V_3(R)$ is linearly dependent.

(ii) Let $T:R^2 \to R^3$ be a linear transformation given by $T(x_1,x_2)=(x_1+x_2,x_1-x_2,x_2)$, then find the rank of T.

4. (i) $W = Span\{x_1,x_2\}$, where $x_1=\begin{bmatrix}3\\6\\0\end{bmatrix},x_2=\begin{bmatrix}1\\2\\2\end{bmatrix}$.Construct orthogonal basis (v_1,v_2) for W

(ii) Let $A=\begin{bmatrix}1&6\\5&2\end{bmatrix},u=\begin{bmatrix}6\\-5\end{bmatrix},v\begin{bmatrix}3\\-2\end{bmatrix}$. Are u and v Eigen vectors of A.

5. (i) Define a linear transformation $T:R^2 \to R^2$ by $T(x)=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-x_2\\x_1\end{bmatrix}$. Find the images under T of $u=\begin{bmatrix}4\\1\end{bmatrix},v\begin{bmatrix}2\\3\end{bmatrix}$ and $u+v=\begin{bmatrix}6\\4\end{bmatrix}$

(ii) Find a vector $x=(c,d)$ that has dot product $x.r=1$ and $x.s=0$ with the given vectors $r=(-2,1),s=(-1,2)$