Module – II
Mechanical Properties of Solids and Fluids

Lectures: 03

Objective:

In this module we will discuss the mechanical properties of solids and fluids. The students will learn different mechanical properties and their uses.

Pretest Questions:

1. In how many states matters exist?
2. What are the differences between solids, liquids and gases?
3. What do you mean by mechanical properties?
4. What are the utility to measure the mechanical properties?
5. Which is more elastic- steel or diamond?
6. Explain why the temperature of a wire under tension will change if it snaps suddenly?
7. Springs are usually made of steel but not of copper. Why?
8. On the basis of moduli of elasticity, distinguish between solid, liquid and gaseous substances.
9. In the case of an elastic body which one is more fundamental –stress or strain?
10. ”Within elastic limit the poison’s ratio depends only on the nature of the material but not on the stress applied”-explain.
11. A steel wire with a greater diameter can withstand a greater load. Why?
12. Can a steel wire be elongated to twice its initial length by hanging a load from its end?

1. Mechanical Properties of Solids

❖ In the general sense, a rigid body is defined as a hard solid object with a definite shape and size. But, from the practical point of view, it is found that rigid bodies can be stretched, compressed and bent when a sufficiently large external force is applied on it. For example, a rigid steel bar can be deformed on the application of suitable force. This means that solid bodies are not perfectly rigid.

➢ **Elasticity**
The property of a body, by which the body opposes any change in its shape and size when the deforming forces are applied and recovers its original state as soon as deforming force is removed, is known as **elasticity** and the deformation caused is known as **elastic deformation**.

➢ **Plasticity**
The property by the virtue of which bodies do not recover its original state as soon as deforming force is removed, is known as **plasticity** and the deformation caused is known as **plastic** deformation.

ELASTIC BEHAVIOUR OF SOLIDS

- Each atom or molecule in a solid is surrounded by atoms or molecules and are bonded together by interatomic or intermolecular forces due to which they stay in a stable equilibrium position.
- Atoms or molecules are displaced from their equilibrium positions on the application of deforming/applied forces.
- On the removal of deforming/applied force interatomic forces drive the atoms or molecules back to their original positions and hence it recovers its original state (i.e. Shape and size). This mechanism is very well visualized by a model of spring-ball system shown in the Fig. 1.1 (Source NCERT book).

If we displace the black ball from its equilibrium position, the restoring force in the spring ball system tries to bring the ball back to its original position. Thus elastic behavior of solid can be visualized via from the microscopic level.

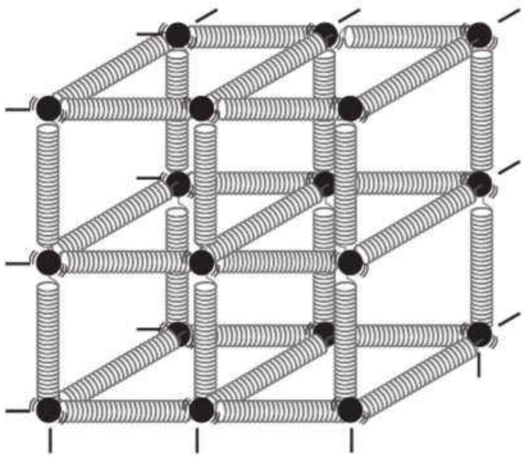


Fig. 1.1 Spring-ball model (Source: NCERT book).

STRESS AND STRAIN

- When we apply a deforming force on the body, a restoring force is developed in the body which is equal in magnitude but opposite in direction to the applied deforming force. This restoring force per unit area is known as stress which is given by

  $$\text{Magnitude of the stress} = F/A \tag{1.1}$$

- The SI unit of stress is Nm^{-2} or Pascal (Pa).

There are three ways {longitudinal (a), shear (b & c) and hydraulic stress (d)} in which a solid may change its dimensions when an external force acts on it. These are shown in Fig. 1.2(a, b, c & d) (Source NCERT book).

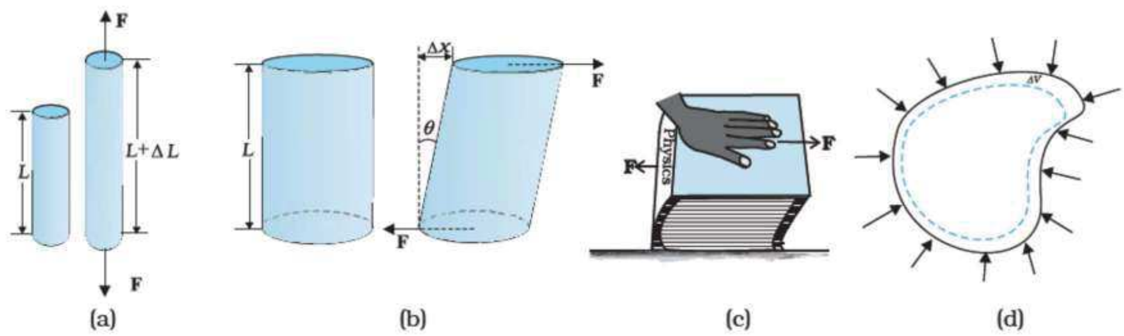


Fig. 1.2 (a) A cylindrical body under tensile stress elongates by ΔL (b) Shearing stress on a cylinder deforming it by an angle θ (c) A body subjected to shearing stress (d) A solid body under a stress normal to the surface at every point (hydraulic stress). The volumetric strain is ΔV/V, but there is no change in shape (Source: NCERT book).

In Fig.1.2 (a), a cylinder is stretched by two equal forces applied normal to its cross-sectional area. The restoring force per unit area in this case is called **tensile stress**. If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as **compressive stress**. Tensile or compressive stress is also known as longitudinal or normal stress.

The normal tensile or compressive stress is expressed as

$$\sigma_{tensile} = \frac{F_{tensile}}{Cross\ \text{sectional area (A)}}$$
$$\sigma_{compressive} = \frac{F_{compressive}}{Cross\ \text{sectional area (A)}} \qquad (1.2)$$

In both the cases, there is a change in the length of the cylinder. The change in the length ΔL to the original length L of the body (cylinder in this case) is known as **longitudinal strain or normal strain** ε.

Longitudinal strain, $\varepsilon = \Delta L / L$ (1.3)

- On the application of applied tangential force, there is a relative displacement Δx between opposite faces of the cylinder as shown in the Fig. 1.2(b). The shear stress (τ) acting on the body is defined as

$$\tau = \frac{F_{\text{tangential}}}{A} \qquad (1.4)$$

- Shearing strain $\Delta x / L = \tan\theta$ (1.5)

- where θ is the angular displacement of the cylinder from the vertical (original position of the cylinder). Usually θ is very small i.e. θ ≤ 10°, tan θ is nearly equal to angle θ (See Fig. 2(b & c)).

$$\text{Thus, shearing strain} = \tan\theta \approx \theta. \quad (1.6)$$

- A solid body when placed in the fluid under high pressure is compressed uniformly on all sides as the force applied by the fluid acts in perpendicular direction on all the points on the surface which decreases its volume without any change of its geometrical shape. This develops internal restoring forces that are equal and opposite to the forces applied by the fluid and is known as hydraulic stress.

The **hydraulic/volume strain** and is defined as the ratio of change in volume (ΔV) of the body to the original volume (V).

$$\text{Volume strain} = \Delta V/V \quad (1.7)$$

HOOKE'S LAW

- Hooke, in 1679, showed experimentally that for small deformations stress and strain are proportional to each other.

$$\text{Stress} \propto \text{Strain}$$
$$\text{Stress} = E \times \text{strain} \quad (1.8)$$

where E is the proportionality constant and is known as 'modulus of elasticity'.

STRESS-STRAIN CURVE

Typical stress strain curve for a metal is shown in Fig. 1.3. This graph is plotted between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced.

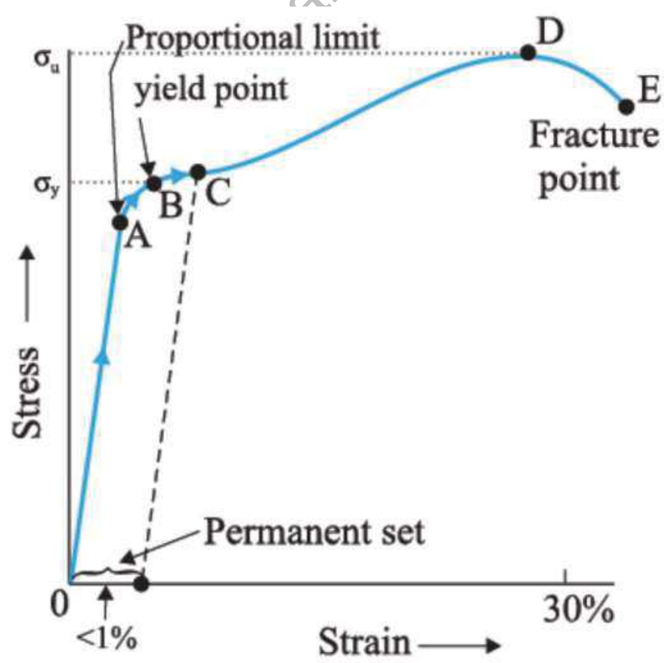


Fig. 1.3 A typical stress-strain curve for a metal (Source: NCERT book).

- The curve is linear in the region between O to A (Hooke's law is obeyed in this region and body behaves as elastic). The relationship between stress and strain in this initial region is not only *linear* but also *proportional.* Beyond point *A*, the proportionality between stress and strain no longer exists; hence the stress at *A* is called the **proportional limit**.

- With an increase in stress beyond the proportional limit, the strain begins to increase more rapidly for each increment in stress. Consequently, the stress-strain curve has a smaller and smaller slope, until, at point *B*, the curve becomes horizontal.

- Point B is the **yield point** (also known as **elastic limit**) and the corresponding stress is **yield strength** (σ_y) of the material.

- Above point B the strain increases rapidly even for a small change in the stress (The portion between B and D). In this region the body does not regain its original dimension and when the stress is made zero, the strain is not zero. Thus the material is said to have a **permanent set** and the deformation is said to be **plastic deformation** (See Fig. 1.3).

- After the point D, additional strain is produced even by a small applied force and fracture occurs at point E (See Fig. 1.3).

- The ratio of stress and strain, in the proportional region within the elastic limit of the stress-strain curve (region OA in Fig. 1.3) is called **modulus of elasticity** and is characteristic of the material.

- It is of great importance to know the elastic limit for applications so that we can avoid the region of plastic deformation which may create problems in designing devices.

Young's Modulus

- The ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ε) is defined as **Young's modulus** and is denoted by the symbol *Y*.

$$Y = \sigma/\varepsilon \tag{1.9}$$

$$Y = (F/A)/(\Delta L/L) = (FL)/(A\Delta L) \tag{1.10}$$

- Dimension of Young's modulus is same as that of stress *i.e.*, Nm^{-2} or Pascal (Pa) as strain is dimensionless quantity.

Table 1.1: Young's moduli, elastic limit and tensile strengths of some materials (Source: NCERT book).

Substance	Young's modulus 10^9 N/m² σ_y	Elastic limit 10^7 N/m² %	Tensile strength 10^7 N/m² σ_u
Aluminium	70	18	20
Copper	120	20	40
Iron (wrought)	190	17	33
Steel	200	30	50
Bone			
(Tensile)	16		12
(Compressive)	9		12

- It is evident from the data of materials given in the Table 1.1 that for metals Young's moduli are large, therefore, they require a large force to produce small change in length.

Question 1: Why steel is more elastic than copper, brass and aluminium?

Solution

Elasticity of a material is also viewed as the resistance offered by the material against the deformation. Higher the resistance offered by the materials against the deformation for a given applied load, higher will be the elasticity of a material. Experimentally, it has been found that to increase the length of a thin steel wire of 0.1 cm² cross-sectional area by 0.1%, a force of 2000 N is required. The force required producing the same strain in aluminum, brass and copper wires with same cross-sectional area are 690 N, 900 N and 1100 N respectively. It indicates that steel offers higher resistance as compared to aluminum, brass and copper for the same amount of strain, hence steel is more elastic than copper, brass and aluminum. Due to the same reason steel is preferred in structural designs.

1.7 SHEAR MODULUS

- The ratio of shear stress to shear strain is called the *shear modulus* of the material (G). It is also called the *modulus of rigidity*.

$$G = \text{shearing stress } (\tau)/\text{shearing strain}$$

$$G = (F/A)/(\Delta x/L)$$

$$= (FL)/(A\Delta x) \qquad (1.12)$$

Using Eq. (1.5)

$$G = (F/A)/\theta$$

$$= F/(A\theta) \qquad (1.13)$$

The shearing stress τ can also be expressed as

$$\tau = G\,\theta \qquad (1.14)$$

SI unit of shear modulus is Nm^{-2} or Pa. The shear moduli of a few common materials are given in Table 1.2.

Table 1.2: Shear moduli *(G)* of some common materials (Source: NCERT book).

Material	G (10^9 Nm^{-2} or GPa)
Aluminium	25
Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6
Nickel	77
Steel	84
Tungsten	150
Wood	10

It is evident from the Table 1.1 and 1.2 that shear modulus (or modulus of rigidity) is in general less than Young's modulus (from Table 1.1 and 1.2).

Bulk Modulus

- The ratio of hydraulic stress to the corresponding hydraulic strain is called *bulk modulus(B)*.

$$B = -p/(\Delta V/V) \tag{1.15}$$

- Negative sign indicates that with an increase in pressure, a decrease in volume occurs.
- SI unit of bulk modulus is the same as that of pressure *i.e.*, Nm^{-2} or Pa.
- The reciprocal of the bulk modulus is called *compressibility* (*k*). It is defined as the fractional change in volume per unit increase in pressure.

$$k = (1/B) = -(1/\Delta p).(\Delta V/V) \tag{1.16}$$

Table 1.3: The bulk moduli of a few common materials (Source: NCERT book)

Material Solids	B (10^9 N m^{-2} or GPa)
Aluminium	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260
Steel	160
Liquids	
Water	2.2
Ethanol	0.9
Carbon disulphide	1.56
Glycerine	4.76
Mercury	25
Gases	
Air (at STP)	1.0×10^{-4}

Question 2: It can be seen from the data given in Table 1.3 that the bulk moduli for solids are much larger than for liquids, which are again much larger than the bulk modulus for gases (air)?

Solution

The bulk modulus of a material is inversely proportional to the compressibility. Since the interatomic bonds between the neighboring atoms in solid are strongest as compared to the liquid and gases, the solids are least compressible. The interatomic bond in the gases are weakest, therefore they are the most compressible substance. Gases are about a million times more compressible than solids. Hence, the bulk modulus of solids is much larger than liquids and gases.

Table 1.4: Summary of stress, strain and various elastic moduli (Source: NCERT book)

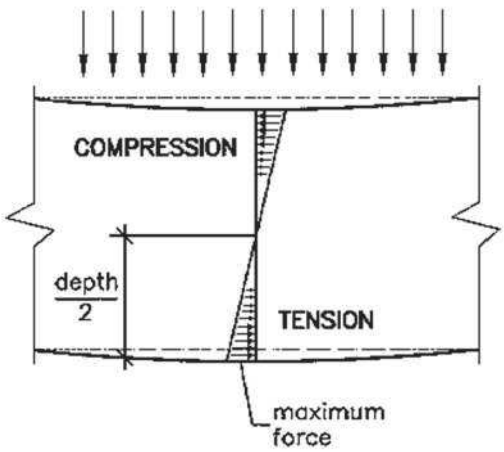
Type of stress	Stress	Strain	Change in shape	Change in volume	Elastic modulus	Name of modulus	State of Mater
Tensile or compressive	Two equal and opposite forces perpendicular to opposite faces ($\sigma = F/A$)	Elongation or compression parallel to force direction ($\Delta L/L$) (longitudinal strain)	Yes	No	$Y = (FL)/(A\,\Delta L)$	Young's modulus	Solid
Shearing	Two equal and opposite forces parallel to oppoiste surfaces [forces in each case such that total force and total torque on the body vanishes ($\sigma_s = F/A$)	Pure shear, θ	Yes	No	$G = (F\theta)/A$	Shear modulus	Solid
Hydraulic	Forces perpendicular everywhere to the surface, force per unit area (pressure) same everywhere.	Volume change (compression or elongation ($\Delta V/V$)	No	Yes	$B = -p/(\Delta V/V)$	Bulk modulus	Solid, liquid and gas

1.9 APPLICATIONS OF ELASTIC BEHAVIOUR OF MATERIALS

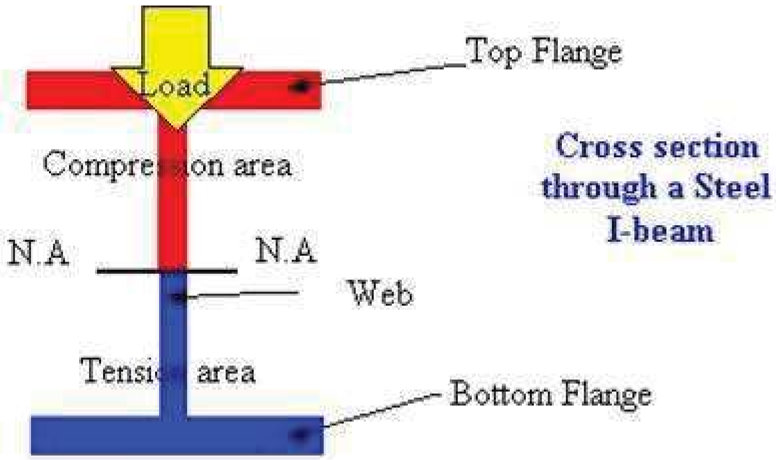
The elastic behavior of materials plays an important role in designing a building, the structural design of the columns, beams and supports. One needs to have the knowledge of the elastic behaviour of materials, so that we do not exceed the limit which can bring trouble.

Question 3: Have you ever thought why the beams used in construction of bridges, as supports etc. have a cross-section of the type I?
Solutions

When a beam bends the top of the beam is in compression and the bottom is in tension.



These forces are greatest at the very top and very bottom. So to make the stiffest beam with the least amount of material you would want the material to be only at the top and bottom sides. However you still need to connect them together or they would just be two separate plates and would not be stiff at all. So you put a web in the middle to connect them and make them work together. The resulting shape is the traditional "I-beam" or wide flange beam.



N.A is a neutral surface area, where the stresses are zero.

Some solved examples:

Question 1: Cranes used for lifting and moving heavy loads from one place to another have a thick metal rope to which the load is attached. The rope is pulled up using pulleys and motors. Suppose we want to make a crane, which has a lifting capacity of 40 tonnes or metric tons (1 metric ton = 1000 kg). How thick should the steel rope be?

Solutions

We obviously want that the load does not deform the rope permanently. Therefore, the extension should not exceed the elastic limit. From Table 1.1, we find that mild steel has yield strength (S_y) of about $300\text{x}10^6$ N m^{-2}. Thus, the area of cross-section (A) of the rope should at least be

$$A \geq W/Sy = Mg/Sy \qquad (1.17)$$
$$= (4\text{x}10^4\text{kg x}10\text{ms}^{-2})/(300\text{x}10^6\text{N m}^{-2})$$
$$= 13.3 \text{ x}10^{-4} \text{ m}^2$$

corresponding to a radius of about 2.06 cm for a rope of circular cross-section. A single wire of this radius would practically be a rigid rod. So the ropes are always made of a number of thin wires braided together, like in pigtails, for ease in manufacture, flexibility and strength.

Question 2: Steel rod of length 2.0 m and radius 15 mm has been stretched along its length by a force of 200 kN. Calculate (a) stress, (b) elongation and (c) strain on the rod. The Young's modulus of steel is 2.0 x 10^{11} Nm^{-2}.

Solution

(a) Stress = F/A = 200 x 10^3 N/3.14 x (15 x 10^{-3}m)^2=2.83 x 10^8 N m^{-2}.
(b) Y = stress/strain= (F/A)/Δl/L
Now Δl (elongation) = (F/A) (L/Y) = [(200x10^3N)x2m] / [3.14x(15x10^{-3})^{2}x2x10^{11}]
2.83mm.
(c) Strain = Δl/L = 2.83/2x10^3 = 1.415.

Question 3: A steel wire of length 6 m and cross-section 5.0 x 10^{-5} m^2 stretches by same amount as a copper wire of length 4 m and cross-section 6.0 x 10^{-5} m^2 under a given load. What is the ratio of the Young's modulus modulus of steel to that of the copper?

Solution

Y = stress/strain= (Mg/A)/Δl/L = Mg L/AΔl
Now let Ys and Yc be the Young's moduli, Ls and Lc the original length's and As and Ac the cross-sectional area of the steel and copper wires respectively. Since, the load Mg and the stretching Δl are same for the two wires respectively. Since, the load Mg and the stretching Δl are same for two wires, we have from the above equation

Ys/Yc=Ls/As x Ac/Lc = 6 m x (6.0 x $10^{-5}$$m^2$) / (5.0 x 10^{-5} m^2) x 4 m = 1.8.

Question 4: Given data:
Initial volume = 200.5 litre, Increase in pressure p = 200.0 atm (1 atm = 1.013 x 10^5 Pa), final volume = 200.0 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Solution
B = volume stress/volume strain = - change in pressure/volume strain = $-p/\Delta v/V$
Here, for water:
p = 200.0 atm = 200.0 x (1.013 x 10^5 Pa) = 2.026 x 10^7 Pa, V = 200.5 litre
and ΔV = 200.0 litre – 200.5 litre = -0.5 litre.
B_{water} = -2.026 x 10^7 Pa/-0.5 litre / 200.5 litre = 8.12 x 10^9 Pa.
The bulk modulus of air at STP is 1.0 x 10^{-4} Pa.
B_{water}/B_{air}= 8.12 x 10^9/1.0 x 10^{-4}=8.12 x 10^{13}.

Question 5: Calculate the pressure required to stop the increase in volume of a copper block when it is heated from 60^0C to 90^0C. Coefficient of linear expansion of copper is α = 8.0 x 10^{-6} per^{0}C and bulk modulus of elasticity is 1.3 x 10^{11} Nm^{-2}.

Solution: Here, we require coefficient of volume expansion of copper (γ) = 3 α (Coefficient of linear expansion of copper.
Now, increase in volume is related to starting volume with coefficient of linear expansion by the relation by relation ΔV = V x γ x (Δt = $t_2 - t_1$)
Volume strain is given by ΔV/V = γ x ($t_2 - t_1$)
Bulk modulus is
B = - change in pressure(p)/volume strain = p/γ(($t_2 - t_1$)
Which gives p = - B γ (($t_2 - t_1$)
On substituting the given values, we have
P = -(1.3 x 10^{11}) x3x 8.0 x 10^{-6} (90-60)
p = 9.36 x 10^7 N m^{-2}.

Question 6: A piece of copper having rectangular cross-section of 13 mm x 18 mm is pulled in tension with 675000 N force, producing only elastic deformation. Calculate the resulting strain. Modulus of rigidity of copper = 4.20 x 10^{10}Pa.

Solution: The modulus of rigidity of the material of the body is given by
η= shearing stress/ shearing strain = (F/A)/θ, where F is the tangential force applied and A the area of cross section of body. Thus shearing strain θ = F/Aη
On substituting the values we get the shearing strain
$\theta = 675000\ N/(2.34 \times 10^{-4} m^2)(4.20 \times 10^{10} N/m^2)$
$\theta = 0.06868$ radians
$\theta = 3.93^0$

2. MECHANICAL PROPERTIES OF FLUIDS

What is Fluid?
Answers
Specifically, a fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude. A shearing stress (force per unit area) is created whenever a tangential force acts on a surface as shown in Fig.1.

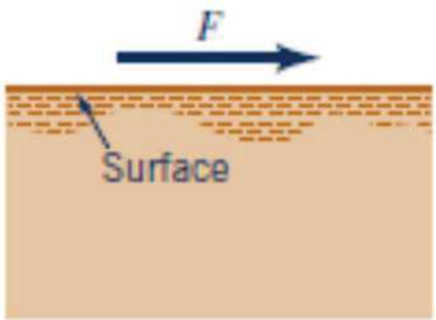


Figure1: Action of shearing force on the surface of liquid
Reference: Munson B.R "Fundamental of Fluid Mechanics" 7th edition, John Wiley & Sons, 2013.

Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied.
Given the definition of a fluid above, there are two classes of fluids, *liquids* and *gases.*

How are fluids different from solids?
We have a general, vague idea of the difference. A solid is "hard" and not easily deformed, whereas a fluid is "soft" and is easily deformed. Although quite descriptive, these casual observations of the differences between solids and fluids are not very satisfactory from a scientific or engineering point of view. A closer look at the molecular structure of materials reveals that matter that we commonly think of as a solid (steel, concrete, etc.) has densely spaced molecules with large intermolecular cohesive forces that allow the solid to maintain its shape, and to not be easily deformed. However, for matter that we normally think of as a liquid (water, oil, etc.), the molecules are spaced farther apart, the intermolecular forces are smaller than for solids, and the molecules have more freedom of movement. Thus, liquids can be easily deformed (but not easily compressed) and can be poured into containers or forced through a tube. Gases (air, oxygen, etc.) have even greater molecular spacing and freedom of motion with negligible cohesive intermolecular forces, and as a