

Fermi Dirac Distribution Function:- The particles which obey Pauli's exclusion principle and are indistinguishable from each other are called Fermions. It has been a well known fact that all half integral spin particles are fermions. Since electrons and holes have half integral spins, therefore these particles are Fermions. ~~In order to~~ The distribution of fermions among various energy levels at a given temperature is governed by a probability distribution function called Fermi-Dirac distribution function, which is given by the following expression:-

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \quad \text{--- ①}$$

This function gives probability that an electron can occupy energy level E at thermal equilibrium. Here E_F is a reference energy called Fermi Energy and energy level corresponding to E_F is called Fermi level. The function $f(E)$ is also known as Fermi Factor. Equation ① is also known as Fermi-Dirac equation.

Variation of Fermi Factor:- The Fermi-Dirac distribution function given in equation ① above can take values only between 0 and 1. Thus it represents probability or occupancy of energy levels by electrons.

Case I At absolute zero temperature (0K)

Consider any energy level corresponding to energy E less than

reference energy E_F (called Fermi-Energy). Therefore $E - E_F$ is negative. Since $T = 0K$ $\therefore$ from ① $f(E) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1+0} = 1$

This means that any level below E_F is ^{completely} occupied with electrons at $0K$.

Now consider an energy-level having energy E greater than E_F . Now $E - E_F$ is positive. Since $T = 0K$ Thus from ① we can write

$$f(E) = \frac{1}{1 + e^{\infty}} = \frac{1}{1+\infty} = 0$$

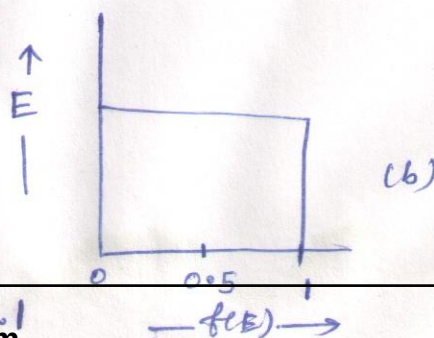
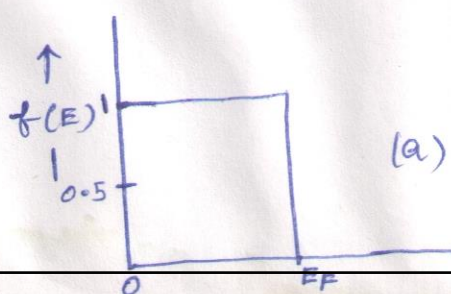
Thus all Energy levels above E_F are completely empty at $0K$.

When $E = E_F$ we take $\lim_{T \rightarrow 0}$ (Rather than $T = 0$) Thus

we get

$$\begin{aligned} f(E_F) &= \lim_{T \rightarrow 0} \frac{1}{1 + e^{(E-E_F)/RT}} \\ &= \lim_{T \rightarrow 0} \frac{1}{1 + e^0} \\ &= \lim_{T \rightarrow 0} \frac{1}{1+1} \\ &= \frac{1}{2} \end{aligned}$$

It means that probability of occupancy of Fermi level is $\frac{1}{2}$. That is, it may be a filled level or an empty level. In fact when E_F is a virtual level (Just like pure/intrinsic semi-conductor), then it is completely empty at $0K$ and when E_F is a real level (just like conductors), then it is completely filled at $0K$. The variation of $f(E)$ at $0K$ for a conductor is shown in Fig (1a) and Fig 1b.



From this discussion, we can say Fermi level at absolute zero -

"It is a level at 0K , below which all existing levels are completely filled with electrons and above which all existing levels are completely empty".

Case II If temperature is more than absolute zero ($T > 0\text{K}$)
When a solid is heated, then electrons start absorbing this heat and transfer to excited states. However the energy given ~~in~~ in this manner can be absorbed by electrons which are near the Fermi level E_F only. The electrons, which are in levels well below E_F are not able to excite to higher states.

Consider a state $E > E_F$, so that $E - E_F$ is positive. Since kT is also positive, therefore if $E \rightarrow \infty$ then $(E - E_F)/kT \rightarrow \infty$

So that $f(E) \rightarrow 0$

On the other hand when $E < E_F$ then $E - E_F$ is negative. But $T > 0 \therefore (E - E_F)/kT$ is also negative. We assume that temp. T is such that quantity kT is much greater than value of E_F . Under this assumption as $E \rightarrow 0$ $(E - E_F)/kT \rightarrow -\infty$
So that ~~$f(E) \rightarrow 0$~~ $f(E) \rightarrow 1$

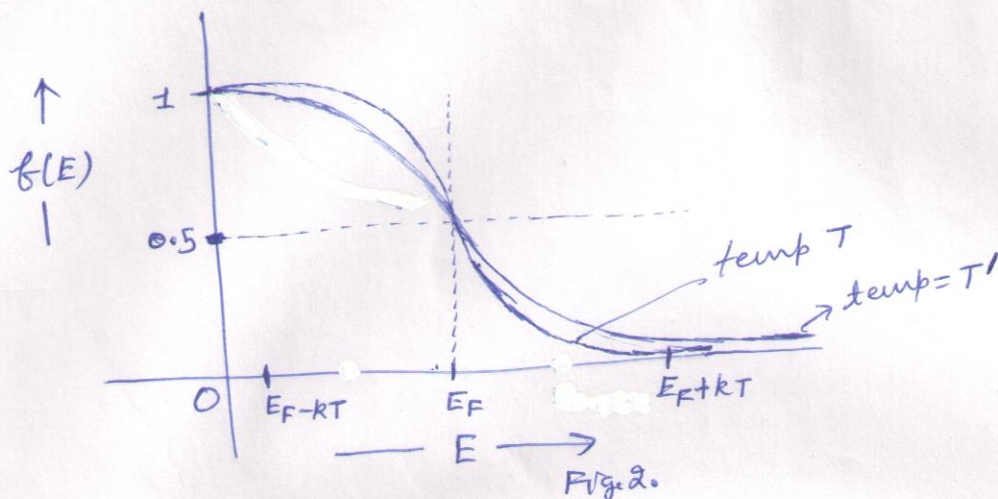
For $E = E_F$ (and $T > 0$ of course!), we get

$$f(E = E_F) = f(E_F) = \frac{1}{1 + e^0} = \frac{1}{2} = 0.5$$

This result is same as in case I. Thus value of Fermi factor ~~$f(E)$~~ $f(E)$ is 0.5 at any temp. whether 0K or any higher temperature.

With this discussion, we can see that shape of graph of $f(E)$ vs E at a given temp should be of the form

as shown in Fig. (2)



Let us now plot same graph at another temperature T' which is greater than T (i.e. $T' > T$). Consider the region $E > E_F$. Since $T' > T \therefore (E - E_F)/kT > (E - E_F)/kT'$

$$\Rightarrow f(E) \text{ at } T < f(E) \text{ at } T' \quad [\text{See equation ①}]$$

~~Now consider~~ Now consider region $E < E_F$. In this case we must remember that $E - E_F$ is negative. Therefore

$$(E - E_F)/kT < (E - E_F)/kT'$$

$$\therefore f(E) \text{ at } T < f(E) \text{ at } T'$$

Thus graph of $f(E)$ at T' will be above graph of $f(E)$ at T in the region $E > E_F$ and it will be below graph of $f(E)$ at T in the region $(E < E_F)$, while $f(E) = \frac{1}{2}$ will be a crossover point as shown in fig 2.

We shall make two changes in Fig. 2 for practical purposes.

- (i) we will show energy along Y-axis and $f(E)$ along X-axis for practical purposes (although it appears to be somewhat illogical right now)
- (ii) Actually tail of graph extends upto ∞ , but for practical purposes

$f(E)$ does not change much in the degenerate region. We will show this graph touching the axes within a region of kT . With these changes, the variation of $f(E)$ at different temperatures (including $T=0K$ case) is shown in fig.3.

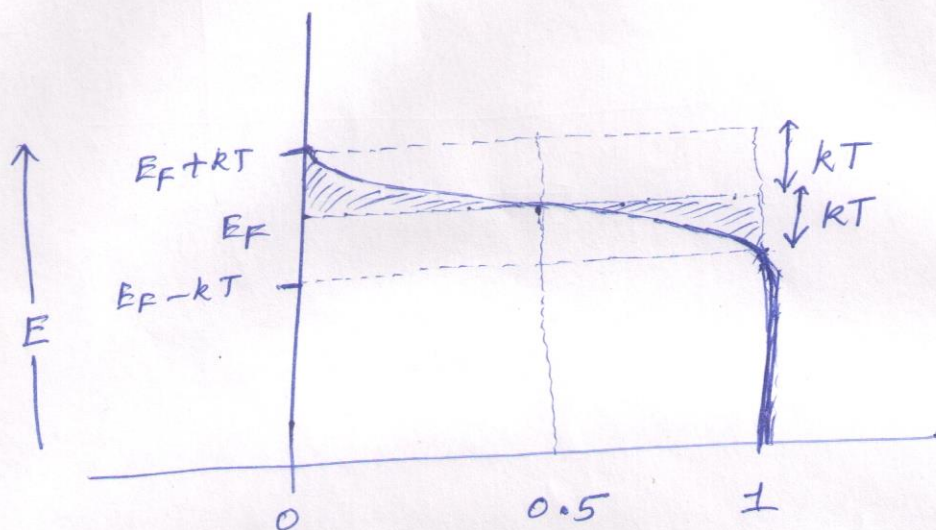


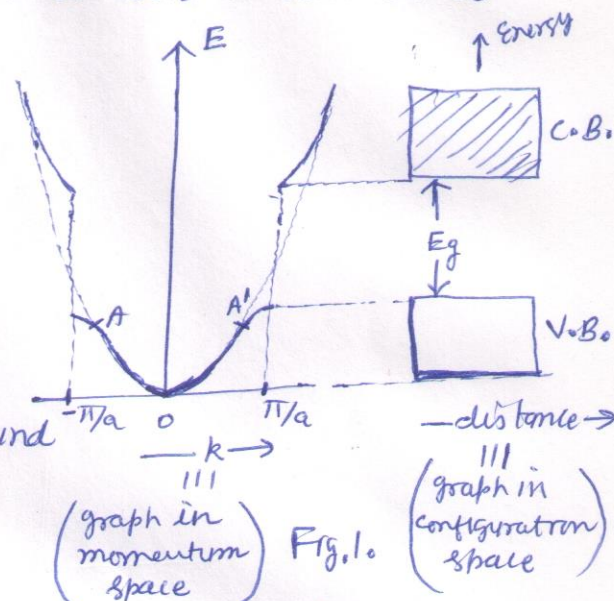
Fig.3

Effective Mass (m^*) :- It is a general perception that mass of an electron in a solid is same as the mass of a free electron. However experimentally, it is observed that ^{inside} some solids the mass of electron, when in motion, is more than the mass of a free electron, while for some other solids, it is less than mass of a free electron. We cannot associate this change in mass with relativistic effects due to two reasons:

- (i) Speed of electron inside a solid is very small as compared to speed of light (Typically of the order of 10^5 m/s) so relativistic effects are supposed to be NIL.
- (ii) Due to relativistic effects mass in motion is always more

while decrease in mass cannot take place. However experimentally decrease in mass is also observed in certain solids.

It is observed that when an electron moves in a solid, it experiences interactions from other electrons and positive ions present in its neighbourhood. The combination of electrons and nearby +ve ions form a potential U around



the moving electron due to which interaction takes place, which results in change in mass of electron moving inside a solid. Thus "Effective mass of an electron is the mass in the presence of lattice potential in a solid". It is denoted by symbol m^* . The expression for effective mass is given by

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} \quad \text{--- (1)}$$

Where $\hbar = \frac{h}{2\pi}$
 $p = \hbar k = \text{linear momentum}$
 $E = \text{Energy of electron moving in a solid.}$

The ^{variation} ~~variation~~ of energy of an electron with respect to propagation constant k in the conduction band and valence band is shown in Fig. 1. Equation (1) can also be written as

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2E}{dk^2} \quad \text{--- (2)}$$

Since $\frac{d^2E}{dk^2}$ represents curvature of graph between E & k . Thus from Fig. 1 above, we can make following conclusions

- (i) Near the bottom of band (V.B.) $m^* \approx m$ (where $m = \text{mass of free electron}$) (because E and k graph is a parabola here, and for free electron potential

energy is zero, so that

$$E = KE + PE$$

$$= \frac{p^2}{2m} + 0$$

$$= \frac{\hbar^2 k^2}{2m}$$

$$\Rightarrow E \propto k^2$$

(Note that for free electron use symbol m for mass and for bound electron use symbol m^* for mass)

which means that E vs k graph of a free electron is a parabola.

(ii) At the points of inflexion A & A' , ~~E vs~~ the graph between E and k changes between concave to convex shape so that $\frac{d^2E}{dk^2} = 0$ $\therefore$ From ② $\frac{1}{m^*} = 0$ or $m^* = \infty$

This means that external potential of lattice cannot exert any action on the motion of electron in this region.

(iii) Between points O & A (or O & A') curvature of E and k curve is +ve so $\frac{1}{m^*}$ is +ve or effective mass of electron is +ve.

(iv) Beyond points A and A' (i.e. near the top of valence band) the E/k graph has negative curvature i.e. $\frac{d^2E}{dk^2}$ ~~is~~ or m^* is negative. Thus effective mass is negative near this region.

The concept of effective mass provides satisfactory description of the charge carriers in crystals. In crystals like alkali metals, energy bands are partially filled and conduction takes place mainly due to electrons. However for crystals where energy band is nearly full, the vacancies having -ve charge & -ve mass are equivalent to particle of +ve charge and +ve mass, called holes.

Consider two narrowly spaced energy levels E and $E+dE$. Let number of electrons existing per unit volume of the sample in this energy range are dn . If density of energy states in this region is $Z(E)dE$ and fermi function is $f(E)$, then we have

$$dn = \left(\frac{\text{Energy states in the region } E \text{ \& } E+dE}{\text{Volume}} \right) \times \left(\text{probability of a state to be filled by an electron} \right)$$

$\downarrow$ density of states $\downarrow$ fermi function

$$= (Z(E)dE) \times f(E)$$

$$\text{or } dn = Z(E)f(E)dE \text{ --- (1)}$$

The expression for density of states in energy region E & $E+dE$ is given by:-

$$Z(E)dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2} dE \text{ --- (2)}$$

Where m_e^* = effective mass of electron in conduction band, E_c = energy corresponding to bottom edge of conduction band.

The expression for fermi function is given by

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \approx e^{-(E-E_F)/kT} \text{ --- (3)}$$

(Neglecting 1 in comparison to exponential term)

For doing this article student has to remember following standard results:-

(i) The density of energy states in conduction band in small energy region dE around energy value E is given by

$$Z(E)dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2} dE$$

$$(ii) \int_0^{\infty} x^{1/2} e^{-ax} dx = \frac{\pi^{1/2}}{2 a^{3/2}}$$

(Well maths is sometimes challenging but always very exciting!)

This integration is challenge to ILATE concept learned in 10+2 classes

Put equations (2) & (3) in (1), we get

$$dn = \left[\frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2} e^{-(E-E_F)/kT} \right] dE$$

$$= \left[\frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2} \times e^{-(E-E_c)/kT} \times e^{-(E_c-E_F)/kT} \right] dE$$

Total number of free electrons per unit volume in the conduct band are given by:

$$n = \int_{E_c}^{\infty} dn = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{-(E_c-E_F)/kT} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_c)/kT} dE \quad \text{--- (4)}$$

In equation (4), we can put $a = \frac{1}{kT}$

and $E - E_c = x$ (say)

So that $dE = dx$

When $E = E_c$ then $x = E - E_c = E_c - E_c = 0$

When $E = \infty$ then $x = \infty - E_c = \infty$

∴ Integration part of equation (4) can be calculated as

$$\int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_c)/kT} dE = \int_0^{\infty} x^{1/2} e^{-ax} dx$$

$$= \frac{\sqrt{\pi}}{2a^{3/2}}$$

$$= \frac{\sqrt{\pi} \times (kT)^{3/2}}{2}$$

Put this value in (4), we get

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{-(E_c - E_F)/kT} \times \frac{\sqrt{\pi} (kT)^{3/2}}{2}$$

$$\Rightarrow n = 2 \left(\frac{2m_e^* \pi kT}{h^2} \right)^{3/2} e^{-(E_c - E_F)/kT} \quad \text{--- (5)}$$

$$\text{or } n = N_c e^{-(E_c - E_F)/kT} \quad \text{--- (6)}$$

$$\text{where } N_c = 2 \left(\frac{2m_e^* \pi kT}{h^2} \right) \quad \text{--- (7)}$$

N_c is called effective density of (energy) states in the conduction band. Equation (5) shows that population density of electrons (n) in the conduction band is a function of temperature and energy of fermi level.

Calculation of hole density (p) in the valence band (VB) of an intrinsic semiconductor:-

$$\text{We know that fermi function } f(E) = \frac{1}{1 + e^{-(E - E_F)/kT}}$$

represents the probability of finding an electron in the energy state E . However a hole is created when an electron is taken out of a bond/band. This ~~means~~ means that presence of hole is equivalent to absence of electron. If $\text{Prob}(h)$ is probability of a hole, then we can write

$$\text{Prob}(h) = 1 - \text{Probability of electron} = 1 - f(E)$$

$$\therefore \text{Prob}(h) = 1 - \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$= \frac{\cancel{1 + e^{(E-E_F)/kT}}}{1 + e^{(E-E_F)/kT}}$$

$$= \frac{e^{(E-E_F)/kT}}{1 + e^{(E-E_F)/kT}} = \frac{1}{e^{-(E-E_F)/kT} + 1}$$

$$= \frac{1}{e^{(E_F-E)/kT} + 1}$$

$$\approx \frac{1}{e^{(E_F-E)/kT}}$$

$$= e^{-(E_F-E)/kT} \quad \text{--- (1)}$$

a state in
 $\therefore$ For Valence band $E < E_F$
 so 1 can be neglected
 in comparison to exponential
 part

The density of energy states within the energy range E and $E+dE$ of valence band is given by:

$$Z(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_v - E)^{1/2} dE \quad \text{--- (2)}$$

(Where m_h^* = effective mass of hole and E_v is energy corresponding to top edge of valence band)

Let dp = concentration of holes per unit volume (also called as hole density) in the energy range E to $E+dE$ within valence band

$$\therefore dp = \text{Prob}(h) \times Z(E)dE$$

$$= e^{-\frac{(E_F - E_V)/kT}{}} \times \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{\frac{1}{2}} dE$$

$$= e^{-\frac{(E_F - E_V)/kT}{}} \times e^{-\frac{(E_V - E)/kT}{}} \times \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{\frac{1}{2}} dE \quad \text{--- (3)}$$

Total number of holes per unit volume in the valence band are given by:

$$p = \int_{-\infty}^{E_V} dp$$

$$= e^{-\frac{(E_F - E_V)/kT}{}} \times \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{\frac{1}{2}} e^{-\frac{(E_V - E)/kT}{}} dE \quad \text{--- (4)}$$

If we put $E_V - E = x$, $\frac{1}{kT} = a$ (say), then

we get $0 - dE = dx$ or $dE = -dx$, when $E = -\infty$ then $x = \infty$

$x = E_V - E = E_V - (-\infty) = +\infty$, when $E = E_V$, then $x = 0$

$\therefore$ Integration part in (4) can be solved as follows

$$\int_{-\infty}^{E_V} (E_V - E)^{\frac{1}{2}} e^{-\frac{(E_V - E)/kT}{}} dE = \int_{\infty}^0 x^{\frac{1}{2}} e^{-ax} (-dx) = \int_0^{\infty} x^{\frac{1}{2}} e^{-ax} dx$$

$$= \frac{\sqrt{\pi}}{2 a^{3/2}} = \frac{\sqrt{\pi} (kT)^{3/2}}{2} \quad \text{--- (5)}$$

$\therefore$ Equation (4) becomes

$$p = e^{-\frac{(E_F - E_V)/kT}{}} \times \frac{4\pi}{h^3} (2m_h^*)^{3/2} \times \frac{\sqrt{\pi} (kT)^{3/2}}{2}$$

$$\text{or } p = 2 \left(\frac{2m_h^* \pi kT}{h^2} \right) e^{-\frac{(E_V - E_F)/kT}{}} \quad \text{--- (6)}$$

or

$$p = N_V e^{-(E_F - E_V)/kT} \quad \text{--- (7)}$$

Where

$$N_V = 2 \left[\frac{2\pi m_k^* kT}{h^2} \right]^{3/2} \quad \text{--- (8)}$$

N_V is called effective density of states in the ~~silicon~~ valence band. Equation (7) shows that hole density/concentration in valence band is also a function of Fermi level E_F and temperature T . It should be noted that N_C in previous article and N_V in the present discussion also depend on temperature T . So to understand dependence of n & p on T these factors (N_C & N_V) cannot be ignored and complete understanding can come only by plotting n & p with temperature.

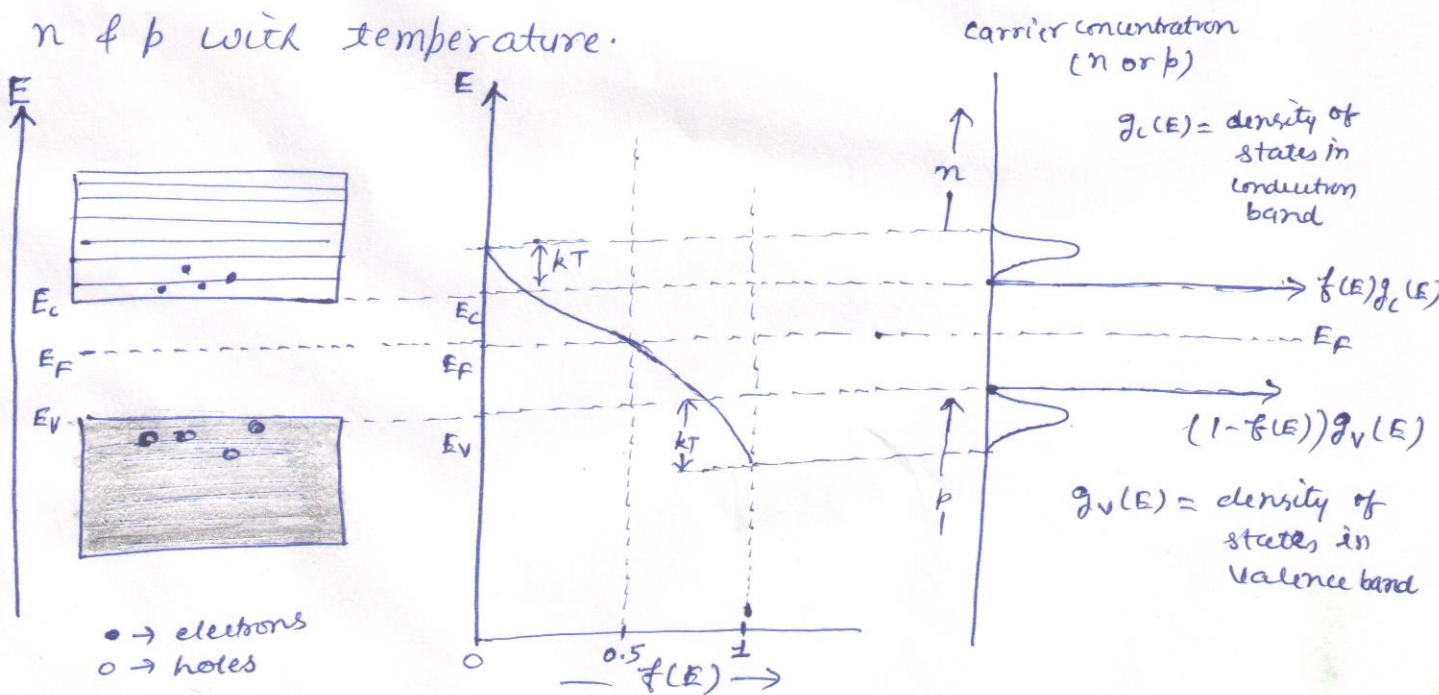


Fig 1.

The Variation of carrier concentration within ~~the~~ conduction and valence band at a given temperature T is shown in figure 1. It is clear that at a given temp. T , electrons are mostly localized in region kT above E_C and holes are localized in small region ~~be~~ kT below E_V .

Intrinsic carrier concentration and its dependence with temperature:-

Whenever a single bond breaks in a semiconductor, then an electron-hole pair (EHP) is generated. Thus at any temperature T , the number of free electrons or holes per unit volume of the sample will be equal. Because of equal value of carrier concentration, we denote it by a general symbol n_i and call it as intrinsic density or intrinsic concentration. Thus if n is concentration of free electrons and p is concentration of holes, then we can write

$$n_i = n \quad \text{--- (1)}$$

$$\text{and } n_i = p \quad \text{--- (2)}$$

$$\text{Multiply (1) \& (2), we get } n_i^2 = np \quad \text{--- (3)}$$

However, expressions of n \& p are given as follows:

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{--- (4)}$$

$$\text{and } p = N_v e^{-(E_F - E_v)/kT} \quad \text{--- (5)}$$

Put values from (4) \& (5) in (3), we get

$$\begin{aligned} n_i^2 &= N_c N_v e^{-(E_c - E_v)/kT} \\ &= N_c N_v e^{-E_g/kT} \quad \text{--- (6)} \end{aligned}$$

$$\text{where } E_g = E_c - E_v \quad \text{--- (7)}$$

E_g represents separation between lowest energy level of conduction band and highest energy level of valence band. It is called band gap energy or simply as band gap.

The values of N_c and N_v are given by following expressions:

$$N_c = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} \times (m_e^*)^{3/2} \quad \text{--- (8)}$$

$$\text{and } N_v = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} \times (m_h^*)^{3/2} \quad \text{--- (9)}$$

Put values from (8) \& (9) in (6), we get

$$\begin{aligned} n_i^2 &= 4 \left(\frac{2\pi kT}{h^2} \right)^3 (m_e^* m_h^*)^{3/2} e^{-E_g/kT} \\ \text{or } n_i &= 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT} \quad \text{--- (10)} \end{aligned}$$

To understand the dependence of n_i on temperature T , we can rewrite equation (10) as follows:

$$n_i = 2 \left(\frac{2\pi k}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} T^{3/2} e^{-E_g/2kT} \quad (11)$$

$$\therefore n_i \propto T^{3/2} e^{-E_g/2kT} \quad (12)$$

From (12), we can draw following conclusions:-

- (i) Intrinsic carrier concentration is independent of the energy of fermi level (E_F)
- (ii) The intrinsic carrier concentration has exponential dependence on band gap energy (E_g) between valence and conduction bands.
- (iii) Intrinsic carrier conc is strongly dependent on temperature.

$$\text{Since } e^{-\{E_g/2kT\}} = 1 - \frac{E_g}{2kT} + \frac{E_g^2}{2! (4k^2T^2)} - \frac{E_g^3}{3! (8k^3T^3)} + \dots$$

$\therefore$ at small temperatures exponential term dominates while at large temperature ($T \rightarrow \infty$) this exponential factor approaches to unit (i.e. as $T \rightarrow \infty$ $e^{-E_g/2kT} \rightarrow 1$)

$\therefore$ at low temperatures we can assume $n_i \propto e^{-\{E_g/2kT\}}$ (from (12))

and at high temperatures we can assume $n_i \propto T^{3/2}$

- (iv) Presence of factor 2 in (12) indicates that two charge carriers (one electron and one hole) are produced when a covalent bond is broken. This process is called production of electron-hole pair (EHP). Thus concentration of electrons in conduction band and concentration of holes in valence band are always equal in an intrinsic semiconductor.

Expression for energy ~~SC~~ of fermi level of an intrinsic

semiconductor: We know that concentration of electrons in conduction band of an intrinsic semiconductor is given as:-

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{--- (1)}$$

and ~~SC~~ concentration of holes in the valence band is given as

$$p = N_v e^{-(E_F - E_v)/kT} \quad \text{--- (2)}$$

$$\text{Where } N_c = 2 \left(\frac{2m_e^* \pi kT}{h^2} \right)^{3/2} \quad \text{--- (3)}$$

$$\text{and } N_v = 2 \left(\frac{2m_h^* \pi kT}{h^2} \right)^{3/2} \quad \text{--- (4)}$$

Since conc. of electrons and holes for intrinsic semiconductor are equal, therefore, we can put

$$\begin{aligned} n &= p \\ \Rightarrow N_c e^{-(E_c - E_F)/kT} &= N_v e^{-(E_F - E_v)/kT} \end{aligned}$$

Taking natural log on both sides, we get-

$$\ln(N_c) - \left(\frac{E_c - E_F}{kT} \right) = \ln(N_v) - \left(\frac{E_F - E_v}{kT} \right) \quad (\because \ln(e) = 1)$$

$$\Rightarrow \ln(N_c) - \ln(N_v) = \left(\frac{E_c - E_F}{kT} \right) - \left(\frac{E_F - E_v}{kT} \right)$$

$$\Rightarrow \ln\left(\frac{N_c}{N_v}\right) = \frac{E_c + E_v - 2E_F}{kT}$$

$$\Rightarrow kT \ln\left(\frac{N_C}{N_V}\right) = E_C + E_V - 2E_F$$

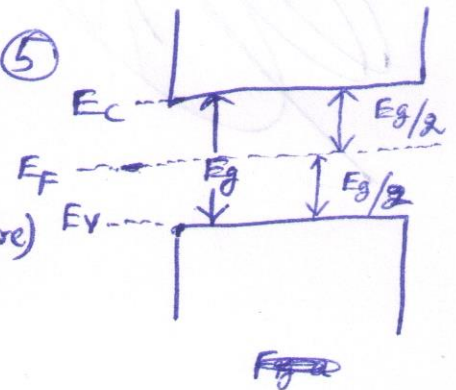
$$\Rightarrow 2E_F = E_C + E_V - kT \ln\left(\frac{m_e^*}{m_h^*}\right)^{3/2} \quad (\text{using (3) \& (4)})$$

$$\Rightarrow \boxed{E_F = \frac{E_C + E_V}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)} \quad \text{--- (5)}$$

Let E_g = band gap energy (as shown in figure here)

∴ From fig., we can write that

$$E_g = E_C - E_V \quad \text{--- (6)}$$



$$\Rightarrow 2E_V + E_g = E_C - E_V + 2E_V$$

or $2E_V + E_g = E_C - E_V$ Put in (5), we get

$$E_F = \frac{2E_V + E_g}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)$$

$$\text{or } \boxed{E_F = E_V + \frac{E_g}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)} \quad \text{--- (7)}$$

If we assume that effective mass of electron and hole are equal (i.e. $m_e^* = m_h^*$), then equation (7) becomes

$$\boxed{E_F \approx E_V + \frac{E_g}{2}} \quad \text{--- (8)}$$

or

$$\boxed{E_F \approx E_C - \frac{E_g}{2}} \quad \text{--- (9)} \quad (\because E_V = E_C - E_g)$$

From (8) & (9), we see that E_F is more than E_V by a factor of $\frac{E_g}{2}$ and it is less than E_C by same factor $\frac{E_g}{2}$. This means that

Fermi level for an intrinsic semiconductor lies in the middle of energy gap

*Note :- Some times energy of valence band top edge is taken as a reference (i.e. $E_v = 0$). In that case we can use the relation

$$E_F = \frac{E_g}{2}$$

Also note that for intrinsic semiconductor, fermi level is not a real level as it lies in middle of energy gap, where no energy state exists.

Variation of Fermi level with temperature in an intrinsic semiconductor :- The expression for Fermi energy in an intrinsic semiconductor is given by:

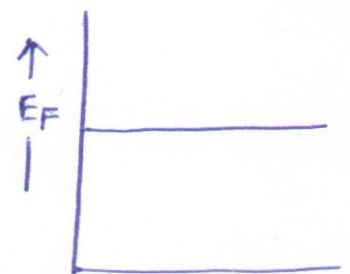
$$E_F = E_v + \frac{E_g}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right) \quad \text{--- (1)}$$

Case I If $m_e^* = m_h^*$

then (1) becomes $E_F = E_v + \frac{E_g}{2}$

$\therefore E_F$ is independent of temperature

The graph between E_F & T is shown in fig(i)

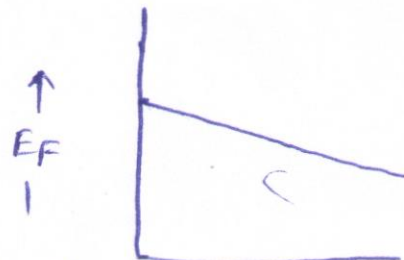


Fig(i)

Case II If $m_e^* > m_h^*$

$\therefore \ln\left(\frac{m_e^*}{m_h^*}\right)$ is positive. Hence from equation (1) we see that E_F decreases

linearly with temperature as shown in Fig(ii).

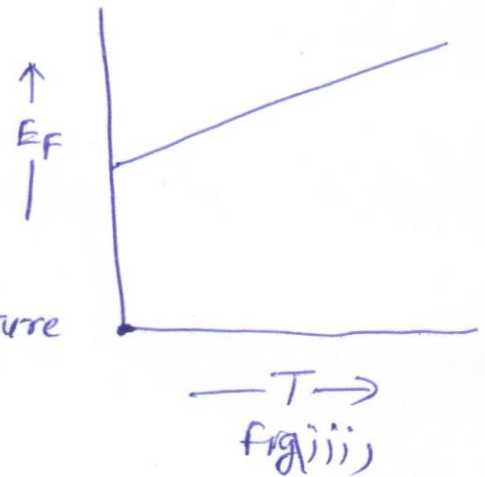


Fig(ii)

Case III If $m_e^* < m_h^*$

In this case $\ln\left(\frac{m_e^*}{m_h^*}\right)$ is negative

$\therefore$ From ① we can conclude that E_F increases linearly with temperature as shown in Fig(iii).



Expression for conductivity of an intrinsic semiconductor and its variation with temperature:-

We know that current in a conductor is related to the drift velocity according to the relation

$$I = nAeV_d \quad \text{--- ①}$$

Where n = concentration of free electrons

A = Area of cross section of metallic conductor

e = ~~may~~ charge on electron

V_d = drift velocity of electrons

In case of semiconductors, both electrons and holes contribute in conduction. Let I_e and I_h are conventional current due to electron and hole in a semiconductor material respectively.

Let A = area of cross section of semiconductor

l = length of semiconductor

e = charge on electron/hole
(magnitude only)

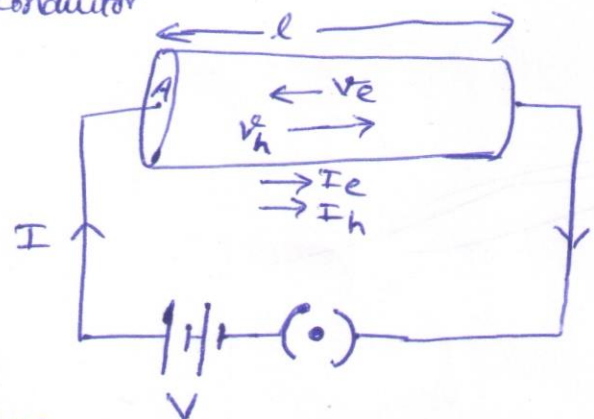
μ_e = electron mobility

μ_h = hole mobility

V = applied potential difference

R = resistance of semiconductor

ρ = resistivity of semiconductor



σ = conductivity of semiconductor

E = applied electric field (by battery)

n = concentration of electrons (free electrons only)

p = concentration of holes

$\therefore$ Using ①, we can write

$$I_e = n A e v_e \text{ --- ②}$$

and $I_h = p A e v_h \text{ --- ③}$

Let I = total conventional current flowing through semiconductor

$$\therefore I = I_e + I_h \quad \left(\because \text{both conventional currents are in same direction} \right)$$

$$= A e (n v_e + p v_h) \text{ --- ④}$$

But electron & hole mobilities are given by following expressions

$$\mu_e = \frac{v_e}{E} ; \quad \mu_h = \frac{v_h}{E}$$

$$\therefore v_e = \mu_e E = \mu_e \frac{V}{l} \quad \left(\because E = \frac{V}{l} \right)$$

$$\text{Hdy } v_h = \frac{\mu_h V}{l}$$

Put in ④ $\therefore I = \frac{V}{l} A e (n \mu_e + p \mu_h)$

$$\therefore I = \frac{V R}{l} A e (n \mu_e + p \mu_h) \quad \left(\because V = I R \right)$$

$$\Rightarrow I = \frac{\rho l}{A} \times \frac{A e}{l} (n \mu_e + p \mu_h) \quad \left(\because R = \frac{\rho l}{A} \right)$$

$$\Rightarrow \frac{1}{\rho} = e (n \mu_e + p \mu_h)$$

or $\boxed{\sigma = e (n \mu_e + p \mu_h)} \text{ --- ⑤ } \left(\because \sigma = \frac{1}{\rho} \right)$

Equation ⑤ gives expression for conductivity of an intrinsic

Semiconductor in terms of mobilities.

But for an intrinsic semiconductor

$$n = p = n_i = \sqrt{N_c N_v} e^{-E_g/2kT} \quad \text{--- (6)}$$

∴ Equation (5) becomes

(See page (14), equation no. (6))

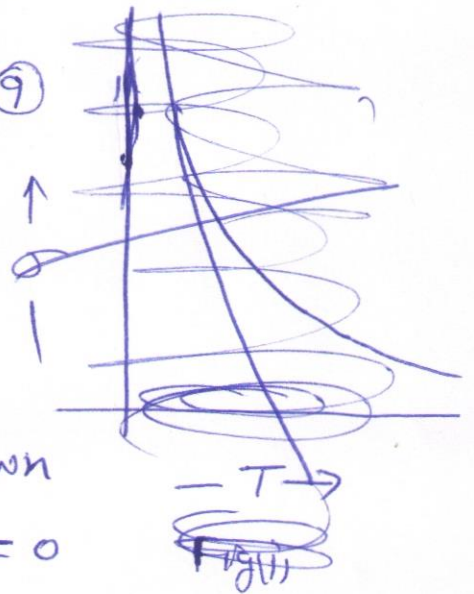
$$\sigma = e(\mu_e + \mu_h) \sqrt{N_c N_v} e^{-E_g/2kT} \quad \text{--- (7)}$$

$$\text{or } \boxed{\sigma = \sigma_0 e^{-E_g/2kT}} \quad \text{--- (8)}$$

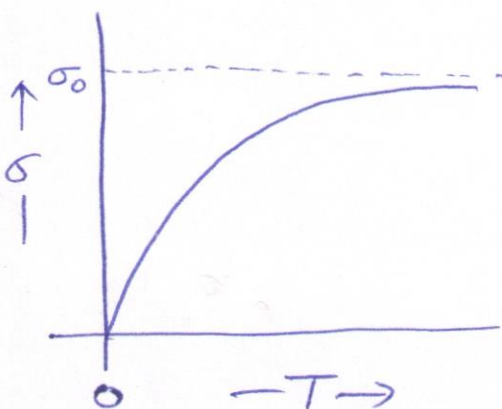
$$\text{Where } \boxed{\sigma_0 = e(\mu_e + \mu_h) \sqrt{N_c N_v}} \quad \text{--- (9)}$$

σ_0 is some constant

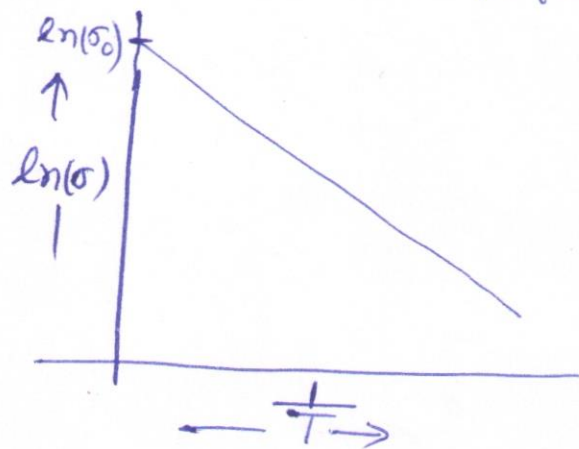
From (8), we see that conductivity of an intrinsic semiconductor increases exponentially with temperature as shown in fig(i) below. Moreover for $T=0, \sigma=0$



∴ at absolute 0K, a semiconductor behaves like perfect insulator.



(Fig(i))



(Fig(ii))

As T is increased σ increases and at $T=\infty, \sigma \rightarrow \sigma_0$. Thus constant σ_0 is maximum conductivity of intrinsic semiconductor,

which it achieves at ∞ Kelvin.

If we take natural logarithm on both sides of (8), we get

$$\ln(\sigma) = \ln(\sigma_0) - \frac{E_g}{2kT} \quad \text{--- (10)}$$

Equation (10) shows that graph between $\ln(\sigma)$ and $\frac{1}{T}$ is a straight line. ~~The slope of~~ This variation is shown in figure above. This relation is very important as it can be used to find the band gap energy of an intrinsic semiconductor. A graph is plotted between $\ln(\sigma)$ & $\frac{1}{T}$ by experimentally measuring conductivity at different values of temperature. Then slope of graph gives $\frac{E_g}{2k}$ value, so that

$$E_g = 2k \times \text{slope of graph}$$

Further Intercept = $\ln(\sigma_0)$

$$\therefore \sigma_0 = e^{\text{intercept}}$$

Since conductivity measurement is relatively difficult than resistance measurement (which is just equal to ratio of instantaneous voltage to current), so E_g is usually measured from plot of logarithm of resistance.

(8) can be written as

$$\frac{1}{\rho} = \frac{1}{\rho_0} e^{-E_g/kT}$$

$$\Rightarrow \rho = \rho_0 e^{E_g/kT} \quad \text{--- (11)}$$

If we put $S = S_0$ in (1), we get $T = \infty$

Thus S_0 is resistivity of semiconductor at ∞ temperature.

Since $R = \frac{\rho l}{A} \Rightarrow S = \frac{RA}{l}$ Put in (1), we get

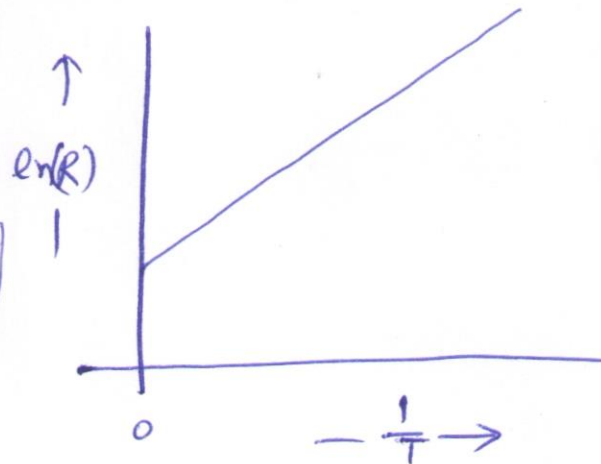
$$\frac{RA}{l} = S_0 e^{\frac{E_g}{2kT}} \Rightarrow R = \frac{S_0 l}{A} e^{\frac{E_g}{2kT}}$$

or $R = R_0 e^{\frac{E_g}{2kT}}$

Where $R_0 = \frac{S_0 l}{A}$

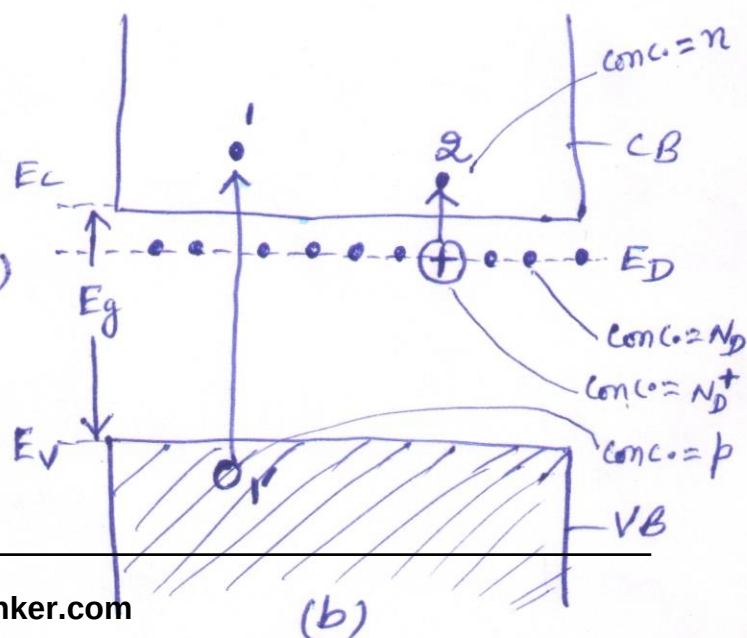
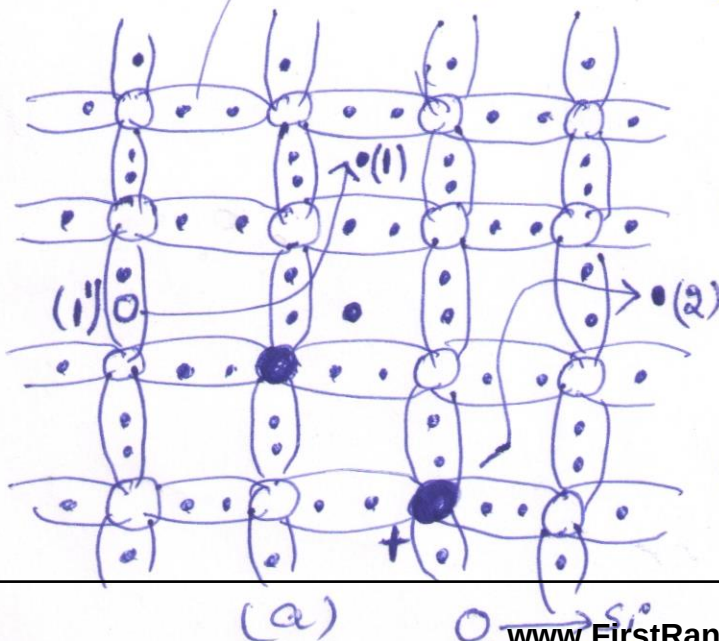
$$\therefore \ln(R) = \ln(R_0) + \frac{E_g}{2kT}$$

$$\therefore E_g = 2k \times \text{slope}$$



Energy band diagram of n-type extrinsic semiconductor and Variation of carrier concentration with temperature:

We know that an n-type extrinsic semiconductor is obtained by adding a pentavalent impurity in semiconductor sample.

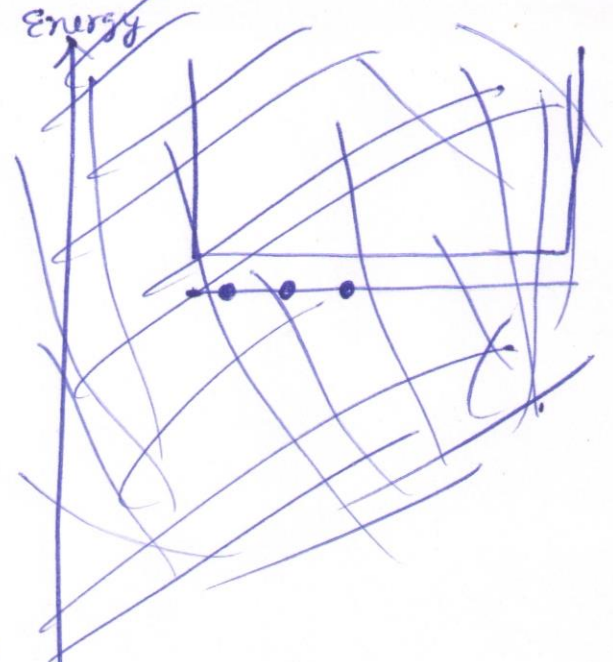


To understand the difference in conductivity behaviour of n-type semiconductor and an intrinsic semiconductor, let us assume that pentavalent impurity atoms, ${}_{15}\text{P}$ are added into ${}_{14}\text{Si}$ semiconductor sample. For ${}_{14}\text{Si}$, $Z=2, 8, 4$, there are four electrons in outermost shell and for ${}_{15}\text{P}$, $Z=2, 8, 5$, there are 5 electrons in outermost shells. The electronic/crystal structure has been shown in 2-D plane in fig (a) above. We observe that all Si-Si ^{covalent} bonds are saturated and each Si atom forms 4 ^{other} covalent bonds with ^{nearest} ~~other~~ Si atoms. All these electrons, which are involved in covalent bond formation are very tightly bound, therefore, cannot take part in conduction. All such electrons form Valence Band (VB), which is shown as shaded region in the band structure (fig(b)). When temp. is absolute zero, then no electron in semiconductor is free, therefore, conduction band is completely empty.

On increasing temperature an electron (like electron (i)) in the covalent bond can come out in the crystal and becomes free. Corresponding to this a +ve vacancy called hole (like hole number (i')) is produced in covalent bond. This transition is shown by vertical arrow $i' \rightarrow i$ in band structure and process is called electron-hole pair (EHP) production. This type of process dominates during conductivity of intrinsic semiconductor and $n=p$ in such cases.

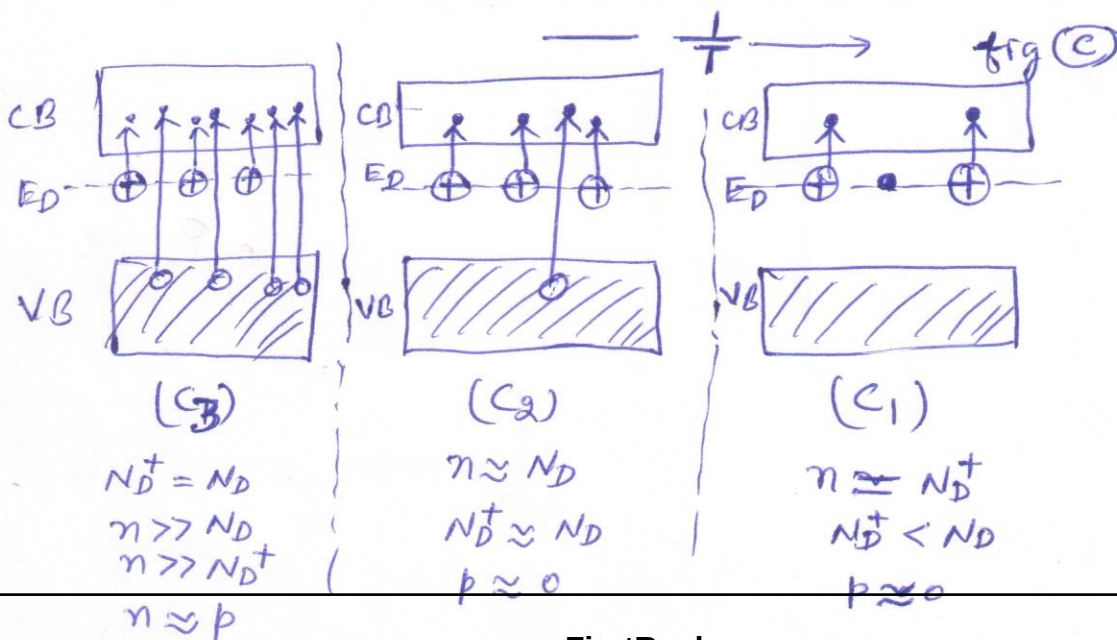
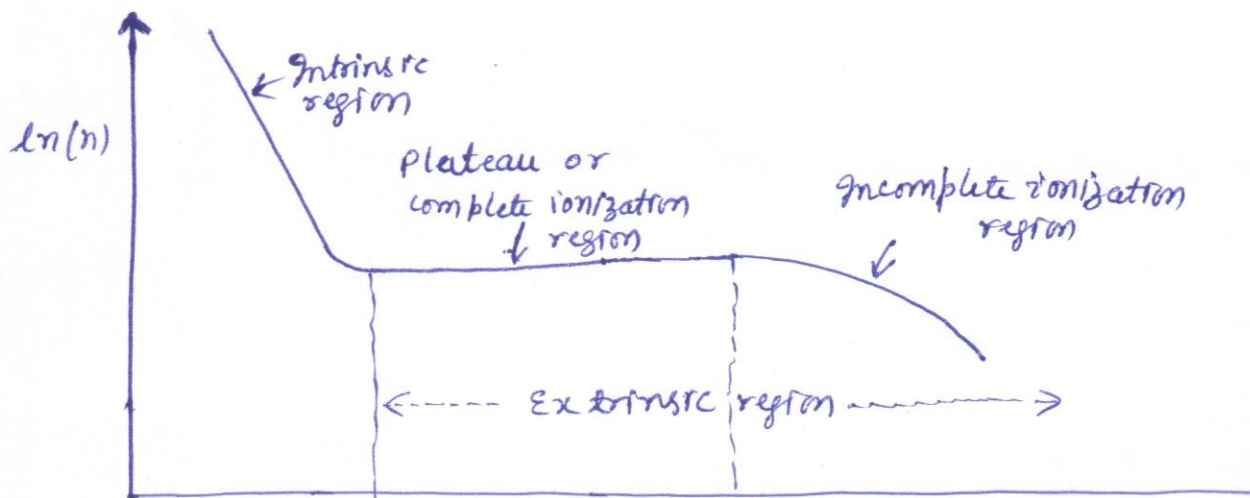
However if we consider impurity atom ${}_{15}\text{P}$ in the

crystal structure (shown as shaded atom), then out of 5 outermost electrons, four are utilized in making covalent bonds with nearest Si atoms and 5th electron cannot do that. This fifth electron is still attached to $15P$ nucleus by relatively less attraction, than other 4 electrons used in forming covalent bonds. This 5th electron is therefore more unstable than other tightly bound electrons and less unstable than completely free electrons (like electron (1)), which lie in conduction band. Thus it is clear that energy of this 5th weakly bound nucleus will be slightly less than that of electrons in CB (i.e. free electrons). Same explanation is applicable for all such 5th electrons of $15P$ impurity atoms. Thus ~~at~~ these 5th e^- s of impurity atoms create a real filled energy level of energy E_D just below the bottom edge of CB. This level, which is created in the energy gap region just below E_C due to addition of pentavalent impurity is called Donor level (shown in fig. (b)).



The difference $E_C - E_D$ is very small as compared to ~~the~~ band gap energy E_g and ~~it can be easily~~ such small energy ($E_C - E_D$) can be easily provided by slightly increasing temperature of n-type semiconductor. When, it is done, then an electron from donor level will ~~then~~ undergo

transition to CB (like electron (2) in fig. (a) & (b)). Since this electron was not a part of any covalent bond before transition. Therefore, this transition cannot create any hole in VB (see fig(a) for better understanding), rather it will create an ~~immobile~~ immobile +ve ion P^+ in the donor level. With this explanation it is clear that for an n-type semiconductor number of free electrons in CB are always more than number of holes in VB (i.e. $n \gg p$). Thus electrons are majority carriers and holes are minority carriers in an n-type semiconductor.



The general dependence of electron concentration on temperature for n-type semiconductor is shown in figure (c) above. At 0 Kelvin ($\frac{1}{T} = \infty$), donor atoms are not ionized, therefore no conduction electron is available and material behaves like perfect insulator. When temp. is slightly increased then donor atoms start ionizing and electrons from E_D start reaching in CB. For each such transition from E_D to CB an electron is created in CB and an ion in E_D . The region is called partial ionization region. In this region conductivity of n-type semiconductor is purely due to electrons in CB and no hole is generated in VB. Moreover as temperature is increased ($\frac{1}{T}$ decreases), conductivity rises sharply.

Let n = conc. of electrons in CB

N_D^+ = conc. of ion generated in Donor level (E_D)

N_D = conc. of donor atoms in Donor level

p = conc. of holes in VB

$\therefore$ at low temp. (partial ionization region) we have

$$n = N_D^+, \quad N_D^+ < N_D, \quad p = 0$$

This region is shown by region C_1 in fig (c)

If we continue to increase temp., a stage is reached where all donor atoms get ionized. With further increase in temp. electron conc. does not rise. This region (C_2) is called plateau or complete ionization region. It is also called Depletion region. In this region carrier conc or conductivity is almost independent of temperature. Moreover

$$n \approx N_D$$

$$N_D^+ = N_D$$

$$p \approx 0$$

Both regions C_1 & C_2 combinedly constitute extrinsic region.

At very high temperature $\left[\left(\frac{1}{T} \rightarrow 0\right), (\text{region } C_3)\right]$ electrons from VB also start transferring to CB and corresponding to each such transfer a hole is generated in VB and this region is called intrinsic behaviour region and EHPs are generated in this region. Obviously $n > N_D$ and $n \approx p = n_i$ and at such very high temperatures material behaves like an intrinsic semiconductor.

Expression for carrier concentration in n-type semiconductor:

Consider an n-type semiconductor in the incomplete ionization region. Let n = conc. of electrons in CB

N_D^+ = conc. of ions in Donor level (E_D)

$N_D = \overset{\text{total}}{\uparrow}$ conc. of donor atoms

N_D^0 = conc. of unionized donor atoms

Since number of holes in VB are negligibly small, therefore, carrier conc. is just approximated by electron conc. in CB.

Moreover, for incomplete ionization region, we can write

$$n = N_D^+$$

$$= N_D - N_D^0$$

$$= N_D - N_D f(E_D)$$

$$= N_D (1 - f(E_D))$$

$$= N_D \left[1 - \frac{1}{1 + e^{(E_D - E_F)/kT}} \right]$$

$$= N_D \left[\frac{e^{(E_D - E_F)/kT}}{1 + e^{(E_D - E_F)/kT}} \right]$$

Where $f(E_D)$ = Fermi function corresponding to energy of donor level E_D at temp. T

$$= N_D \left[\frac{1}{e^{\frac{(E_D - E_F)}{kT}} + 1} \right]$$

$$= N_D \left[\frac{1}{e^{\frac{-(E_D - E_F)}{kT}} + 1} \right] \quad \text{--- ①}$$

However E_F is a few kT times above E_D $\therefore e^{\frac{-(E_D - E_F)}{kT}} \gg 1$

Hence eq. ① can be approximated as

$$n \approx N_D e^{\frac{(E_D - E_F)}{kT}} \quad \text{--- ②}$$

But the electron conc. in CB is given by

$$n = N_C e^{\frac{-(E_C - E_F)}{kT}} \quad \text{--- ③}$$

comparing ① & ②, we get

$$N_D e^{\frac{(E_D - E_F)}{kT}} = N_C e^{\frac{-(E_C - E_F)}{kT}}$$

Take natural log on both sides, we get

$$\ln N_D + \frac{(E_D - E_F)}{kT} = \ln N_C - \frac{(E_C - E_F)}{kT}$$

$$\therefore \ln \left(\frac{N_C}{N_D} \right) = \frac{E_D - E_F + E_C - E_F}{kT}$$

$$\therefore 2E_F = E_D + E_C - kT \ln \left(\frac{N_C}{N_D} \right)$$

$$\Rightarrow E_F = \frac{E_D + E_C}{2} + \frac{kT}{2} \ln \left[\frac{N_D}{N_C} \right]$$

$$\Rightarrow E_F = \frac{E_D + E_C}{2} + \frac{kT}{2} \ln \left(\frac{N_D}{N_C} \right) \quad \text{--- (4)}$$

Equation (4) gives expression for fermi energy of an n-type semiconductor.

We can further modify equation (4) as follows:-

$$\begin{aligned} E_F &= \frac{E_D - E_C + 2E_C}{2} + \frac{kT}{2} \ln \left(\frac{N_D}{N_C} \right) \\ &= \frac{E_D - E_C}{2} + E_C + \frac{kT}{2} \ln \left(\frac{N_D}{N_C} \right) \end{aligned}$$

$$\Rightarrow \frac{E_F - E_C}{kT} = \frac{E_D - E_C}{2kT} + \frac{1}{2} \ln \left(\frac{N_D}{N_C} \right) \quad \text{(dividing both sides with } kT \text{)}$$

$$\text{or } \frac{E_F - E_C}{kT} = \frac{E_D - E_C}{2kT} + \ln \left(\sqrt{\frac{N_D}{N_C}} \right) \quad \text{--- (5)}$$

Now expression for conc. of electrons is given by

$$\begin{aligned} n &= N_C e^{-(E_C - E_F)/kT} = N_C e^{(E_F - E_C)/kT} \\ &= N_C e^{\left[\frac{E_D - E_C}{2kT} + \ln \left(\sqrt{\frac{N_D}{N_C}} \right) \right]} \quad \text{(using (5))} \end{aligned}$$

$$= N_C \left(e^{\frac{E_D - E_C}{2kT}} \right) \times e^{\ln \sqrt{\frac{N_D}{N_C}}}$$

$$\Rightarrow n = N_C \times \sqrt{\frac{N_D}{N_C}} \times e^{\frac{E_D - E_C}{2kT}}$$

$$\text{or } n = \sqrt{N_C N_D} e^{\frac{(E_D - E_C)/2kT}{}} \quad \text{--- (6)}$$

SC

We know that $N_c = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2}$ — (7)

If we use this value of N_c in (4) and (6), then expression for conc. of majority carriers (i.e. 'n') in n-type semiconductor and value of fermi energy E_f are given as follows: -

$$(4) \Rightarrow E_f = \frac{E_D + E_C}{2} + \frac{kT}{2} \ln \left(\frac{N_D}{2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2}} \right) \quad \text{--- (8)}$$

f (6) $\Rightarrow n = N_D \times 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2}$

$$f (6) \Rightarrow n = \sqrt{2 N_D} \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/4} e^{\frac{E_D - E_C}{2kT}} \quad \text{--- (9)}$$

Equation (8) gives expression for Fermi Energy of n-type semiconductors in terms of temperature and equation (9) gives expression for concentration of majority carriers (i.e. electrons) for n-type semiconductors in terms of temperature.

Case - I Variation of Fermi level with temperature ~~with~~ in an n-type semiconductor

In equation (8), if we put $T=0$ [as $T \rightarrow 0$, $kT \rightarrow 0$ faster

and $\ln \left(\frac{N_D}{2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2}} \right) \rightarrow \ln(\infty)$ slower $\therefore \frac{kT}{2} \ln \left(\frac{N_D}{2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2}} \right) \rightarrow 0$

then equation (8) gives us the result

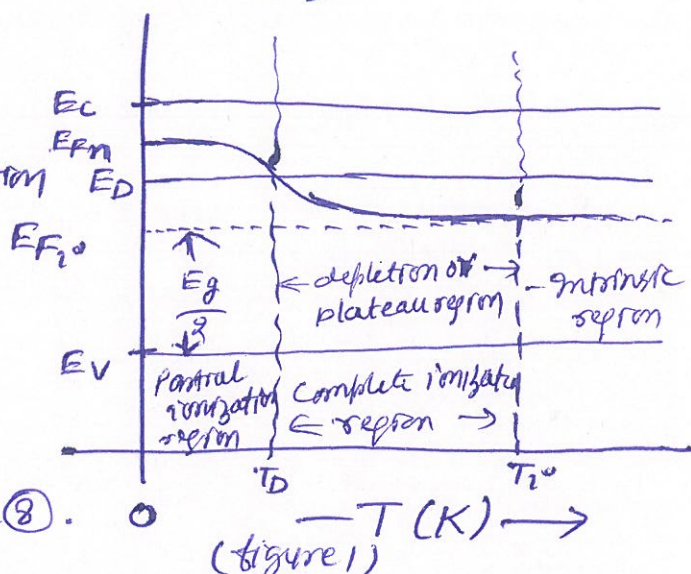
$$E_{Fn} = \frac{E_D + E_C}{2}$$

(i.e. E_f is equal to arithmetic mean of E_D & E_C)
(E_f is now denoted by E_{Fn} just to remind that discussion is for n-type semiconductor)
Thus E_{Fn} lies in the middle

of gap b/w energies of Donor level and lower edge of conduction level.

As the temperature is increased above 0 K, then E_{Fn} starts

decreasing according to equation (8).



(figure 1)

This is due to the fact that donor atoms in donor levels start losing electrons and donor level start becoming more and more empty (Remember that for $T > 0$ K, some states above Fermi may get filled, while, some states below Fermi may be empty). This variation of E_{Fn} continues upto a temperature T_D called depletion temperature. At this temperature all donor atoms get completely ionised and it thus indicates the end of partial ionization region ~~and depletion region~~. At this temp, E_{Fn} matches with E_D .

If temperature is further increased, then E_{Fn} becomes smaller than E_D and complete ionization / plateau region starts. This process of lowering of E_{Fn} ~~with time~~ below E_D with increase in temperature continues upto a temperature T_i called intrinsic temperature. At this temperature, value of E_{Fn} reaches middle of energies of E_C & E_V and it becomes identical with

fermi

↑ energy of an intrinsic semiconductor (E_{Fi}). If temperature increased more than T_i , then E_{Fn} remains constant thereafter.

This Variation of E_F with temperature for n-type semiconductor is shown in figure 1 above.

Case II Variation of majority carrier concentration with temp. for n-type semiconductor.

This Variation of n versus $\frac{1}{T}$ has already been discussed and graph as been plotted on page (26) of these notes.

Case III Law of mass action for n-type semiconductor

Let us consider intrinsic semiconductor first. In such a case, the electron and hole concentrations are equal i.e.

$$n = p = n_i \quad (\text{say}) \quad \text{--- (10)}$$

However, expression for n and p for intrinsic semiconductor are given by

$$n = N_C e^{-(E_C - E_F)/kT} \quad \text{--- (11)}$$

$$p = N_V e^{-(E_F - E_V)/kT} \quad \text{--- (12)}$$

so, that $n \times p = N_C e^{-(E_C - E_F)/kT} \times N_V e^{-(E_F - E_V)/kT}$

$$\Rightarrow n_i \times n_i = N_C N_V e^{-(E_C - E_F + E_F - E_V)/kT} \quad \text{(using (10))}$$

$$\text{or } n_i^2 = N_C N_V e^{-(E_C - E_V)/kT}$$

$$\therefore n_i^2 = N_C N_V e^{-E_g/kT} \quad \text{--- (13)} \quad (\because E_C - E_V = E_g)$$

we know that $N_C = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2}$
and $N_V = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2}$ } --- (14)

Equations (14) show that if we keep T constant, then N_C, N_V remain constant. We know that extrinsic semiconductor is obtained by adding impurity to intrinsic semiconductor. This means, at a given temp., value of N_C & N_V will be same for extrinsic semiconductor. Therefore if we use symbols n_n & p_n for conc. of electrons and conc. of holes for an n-type semiconductor at a given temp., then their expressions will be very similar to (11) & (12), with only difference that E_F must be replaced by E_{Fn} , because fermi energy of ntype and an intrinsic semiconductor are not same (see equation (8) in this article) i.e.

$$n_n = N_C e^{-(E_C - E_{Fn})/kT} \quad (15)$$

$$p_n = N_V e^{-(E_{Fn} - E_V)/kT} \quad (16)$$

If we multiply equations (15) & (16), we get

$$\begin{aligned} n_n \times p_n &= N_C \times N_V e^{-(E_C - E_{Fn} + E_{Fn} - E_V)/kT} \\ &= N_C \times N_V e^{-(E_C - E_V)/kT} \\ &= N_C \times N_V e^{-E_g/kT} \quad (17) \quad (\because E_C - E_V = E_g) \end{aligned}$$

From (13) and (17) we get

$$\boxed{n_n \times p_n = n_i^2} \quad (18)$$

Equation (18) is called law of mass action. It states that

product of majority and minority carrier concentration in an extrinsic semiconductor at a particular temperature is constant and is equal to square of intrinsic carrier concentration at that temperature. This law is very important for calculating minority carrier concentration in an extrinsic semiconductor.

Note:- For p-type semiconductor law of mass action is written as $n_p \times p_p = n_i^2$

where n_p = conc. of electrons in p-type semiconductors (minority carrier conc.)

& p_p = conc. of holes in p-type semiconductors (majority carrier conc.)

Case IV charge neutrality condition for n-type semiconductor

Figure (2) shows energy band diagram of n-type semiconductor under the assumption of complete ionization. i.e. all donor atoms in E_D have already lost their 5th electron due to increased temperature. If more energy is given then a few electrons from

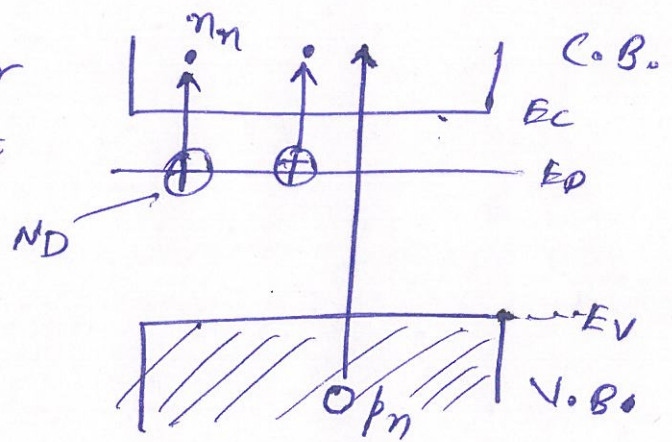


figure (2)

V.B. can migrate to C.B. This will generate electron-hole pairs (EHPs). However no transition from $E_D \rightarrow C.B.$ will produce any hole. From this diagram and explanation given here, it is clear that total number of free electrons in C.B. will be equal to total no. of donor atoms (N_D) plus total number of holes generated in V.B. (p_n). i.e.

$$n_n = N_D + p_n \quad \text{--- (19)} \quad \text{or} \quad n_n - p_n = N_D \quad \text{--- (20)}$$

However for n-type semiconductor $n_n \gg p_n$. Thus equation (20) can be approximated as

$$\boxed{n_n \approx N_D} \quad \text{--- (21)}$$

Equation (3) is called charge neutrality condition for n-type semiconductor.

Case V calculation of minority carrier conc. for n-type semiconductor

If we put value of n_n from (21) in (18), we get

$$N_D \times p_n = n_i^2$$

$$\Rightarrow \boxed{p_n = \frac{n_i^2}{N_D}} \quad \text{--- (22)}$$

For p-type semiconductor, one can prove that

$$n_p = \frac{n_i^2}{N_A}$$

Equation (22) gives expression for minority carrier concentration of n-type semiconductor.

Case VI Variation of Fermi level of n-type semiconductor with impurity concentration (keeping temp. constant)

We all know that when we add impurity atoms from group -V, then a donor level is formed just below E_c for n-type semiconductor and expression for E_F of n-type semiconductor is given by equation number (4) or (8).

If temp. is kept constant (except OK), then equation (8) shows that

E_F is a function of N_D . That is Fermi level of n-type semiconductor

will change with change in conc. of impurity atoms (N_D).

This variation of E_{F_n} with conc. of Donor atoms for an n-type semiconductor is shown in figure 3 here.

At low impurity conc. donor level is a sharp energy state and E_{F_n} is in the middle of energy gap b/w E_c & E_D .

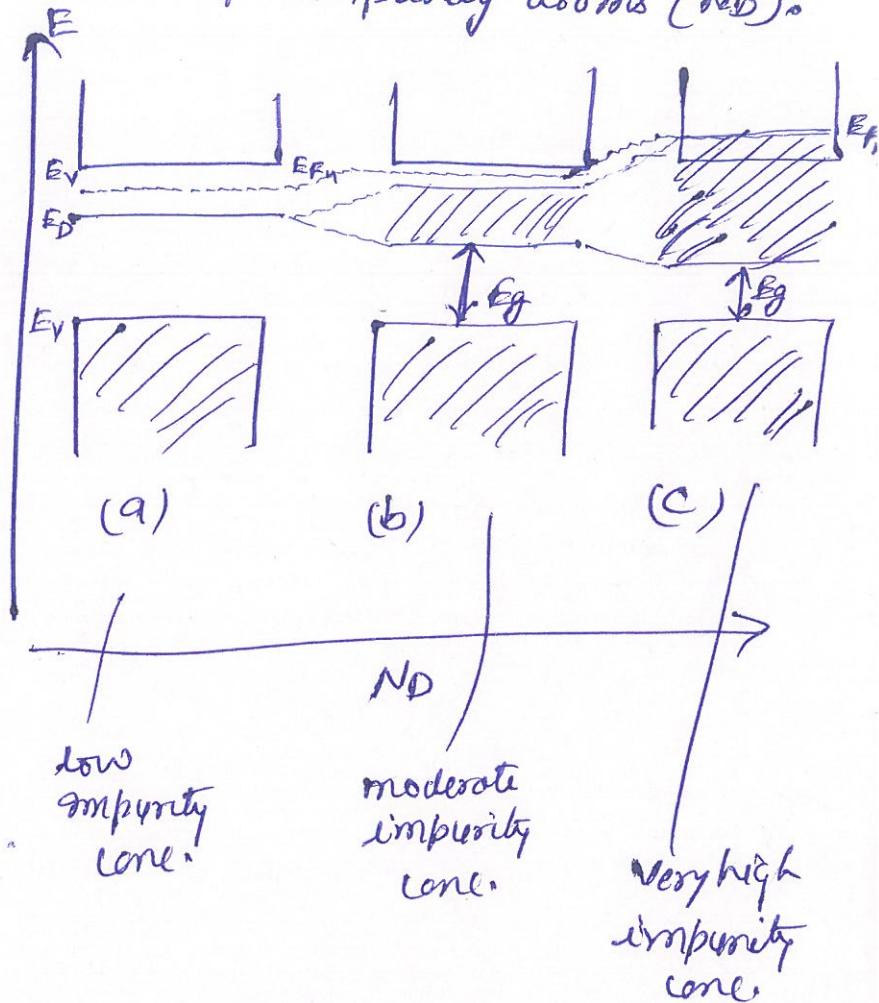


Figure (3)

When conc. of impurity atoms (i.e. N_D) is moderately increased, then value of energies of electron attached to Donor atom (i.e. value of E_D) starts spreading over a narrow energy region due to increase in interactions among donor atoms at ~~low~~ increased impurity conc. level. Due to energy spread, donor level becomes donor band and value of forbidden energy gap E_g is decreased in the result.

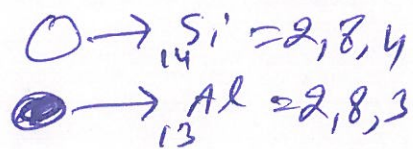
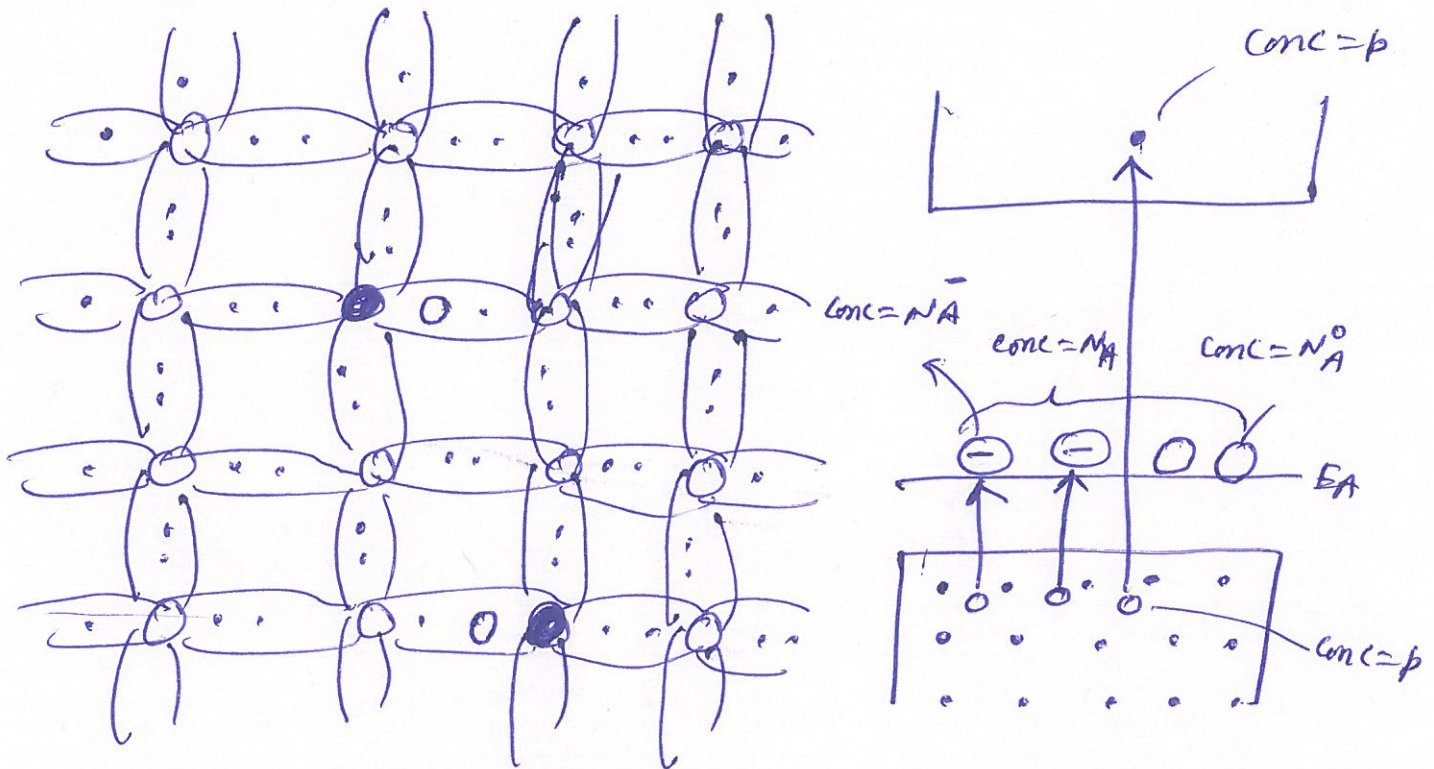
When impurity concentration becomes very high, then interactions among donor atoms become very strong leading to enormous energy spread of E_D so and so - that donor band enters conduction band ~~and so Fermi level~~ so that Fermi level

E_{Fn} is the top most level in spreaded donor band & as a result E_{Fn} is inside CB and E_g is further reduced due to energy spread of E_D .

~~Expression for~~

Energy band diagram of p-type extrinsic semiconductor and Expression for E_f & majority carrier conc. :-

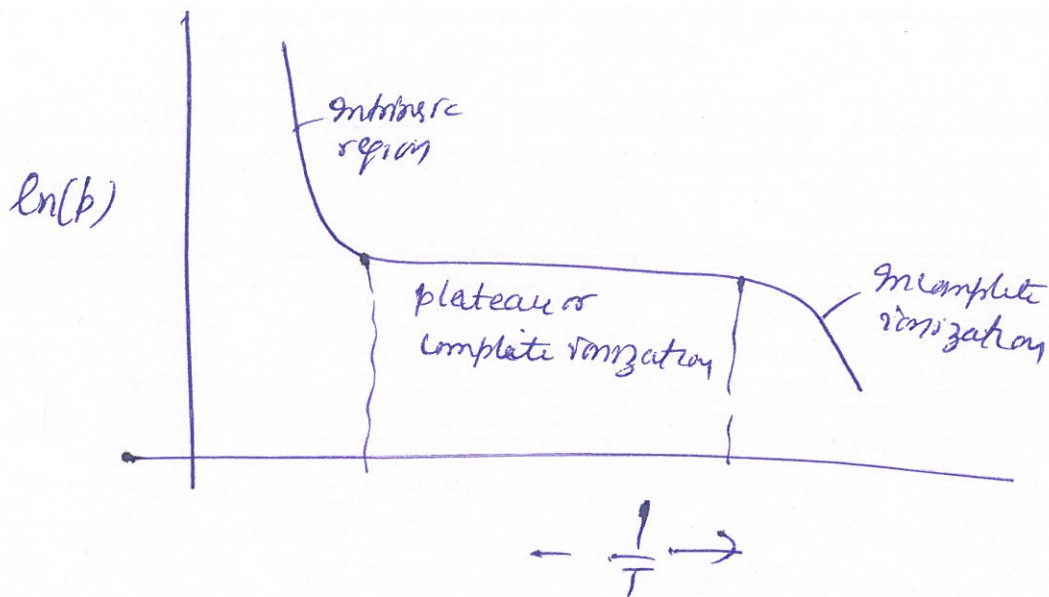
This part is self study for the reader. However final results are given here:-



$$N_A = N_A^- + N_A^0$$

$n \ll p$
 / \
 minority majority

Give explanation yourself



$$\begin{aligned}
 p &= N_A - N_A^0 \\
 &= N_A - N_A \times [1 - f(E_A)] \\
 &= N_A f(E_A) \\
 &= N_A e^{-(E_A - E_F)/kT}
 \end{aligned}$$

$$E_F = \frac{E_V + E_A}{2} + \frac{kT}{2} \ln \left(\frac{N_V}{N_A} \right)$$

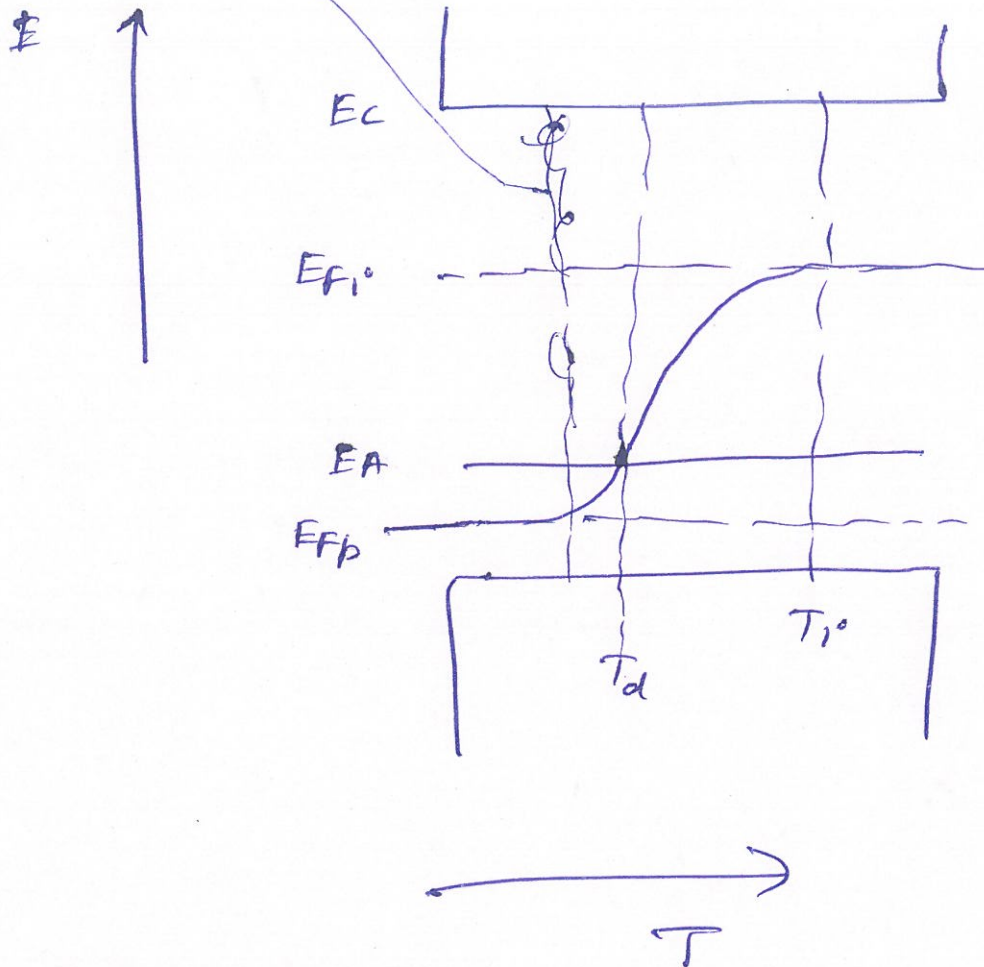
or

$$p = \sqrt{2N_A} \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/4} e^{(E_V - E_A)/2kT}$$

f

$$E_F = \frac{E_V + E_A}{2} - \frac{kT}{2} \left(2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \frac{N_A}{p} \right)$$

Case B



$$n_p \times p_p = n_i^2 \quad \text{— law of mass action}$$

$$p_p = N_A + n_p$$

$$p_p - n_p = N_A$$

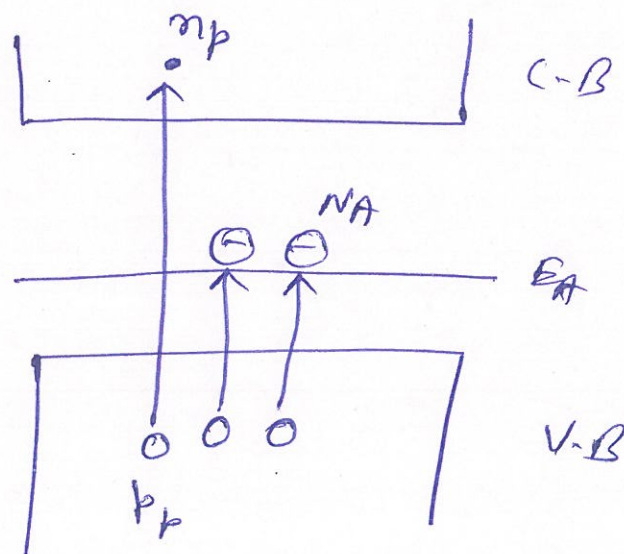
$$(But \quad p_p \gg n_p)$$

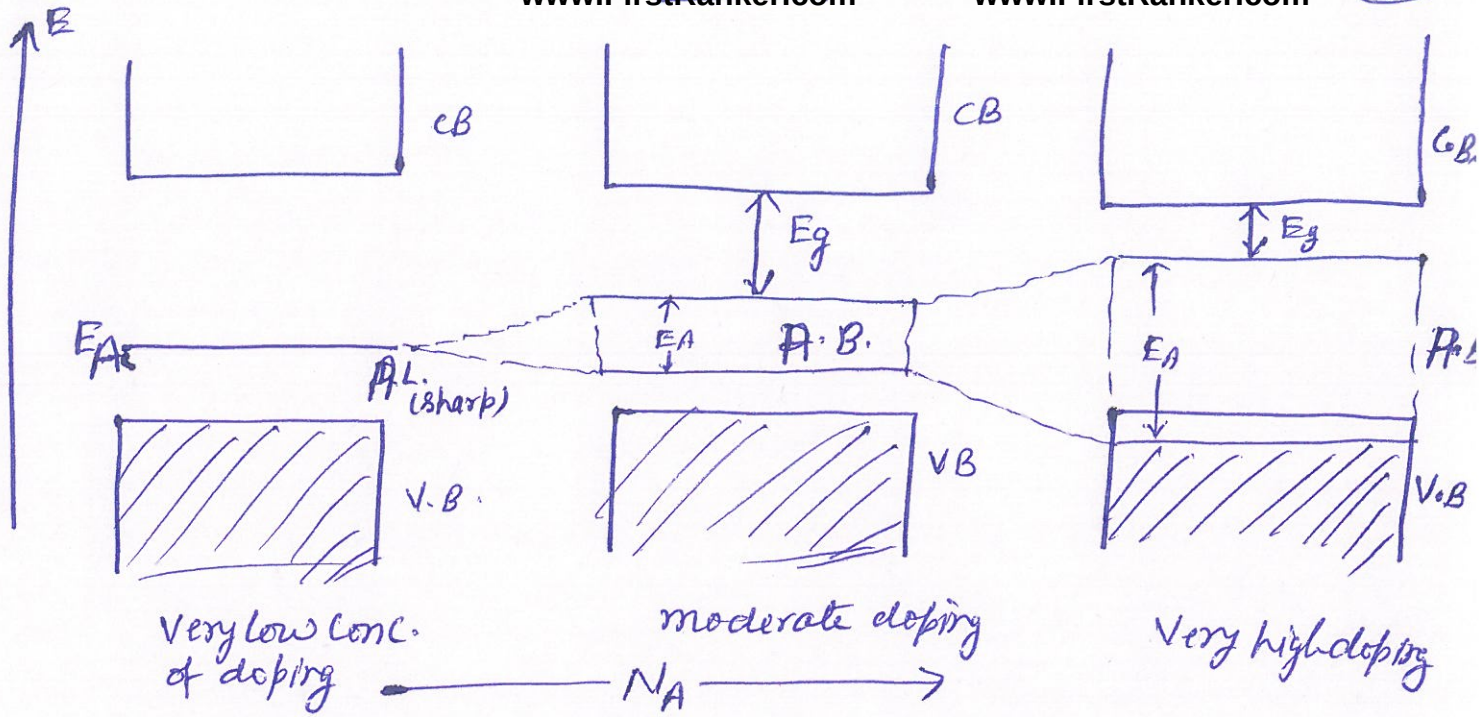
$$\therefore p_p - 0 \approx N_A$$

$$\Rightarrow p_p \approx N_A$$

charge neutrality condition

$$n_p = \frac{n_i^2}{N_A} \quad \text{— minority carrier conc. formula}$$





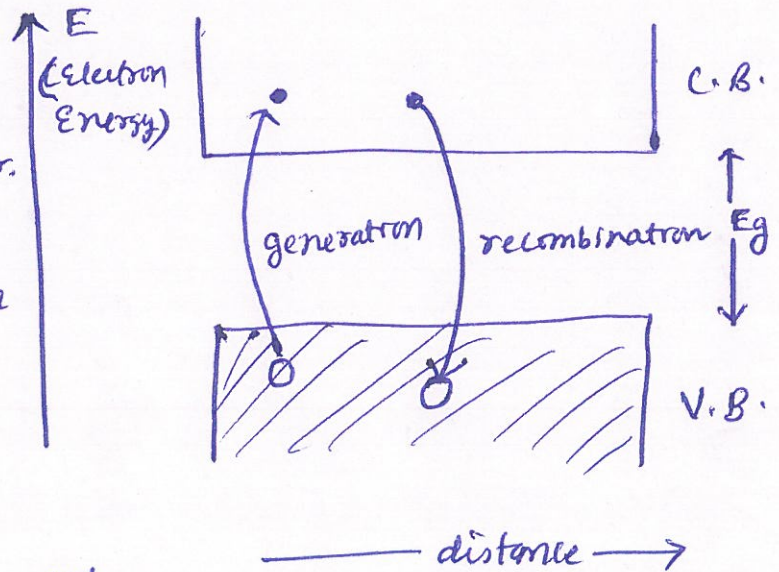
Explain things Yourself

Here; $A.L. \rightarrow$ Acceptor level, $A.B. \rightarrow$ Acceptor Band
 $V.B. \rightarrow$ valence band, $CB \rightarrow$ conduction band
 $E_A \rightarrow$ energy of acceptor level/band
 Note that acceptor level/band is an empty level/band

Carrier Generation & Recombination:-

carrier generation:-

Figure given here shows ~~band~~ energy band diagram of an intrinsic semiconductor. When a single covalent bond breaks, it leads to generation of two charge carriers, one is electron, which goes to $C.B.$ and second is hole, which remains in $V.B.$ (This is already



discussed for n-type semiconductor at page number (23) of notes and this process is shown as $i' \rightarrow i$ transition in crystal structure as well as energy band diagram). This process is called electron-

hole pair production (EHP). This process requires an energy greater than E_g to be available near the bond. This required energy can be given in two ways (i) by ~~giving~~ increasing temperature of semiconductor (This energy is called thermal energy & process of generation of EHP in this case is called thermal generation of carriers) (ii) by irradiating semiconductor surface with beam of electromagnetic radiation of suitable frequency and intensity (This energy is called optical energy and process is called optical generation). Thus process of EHP generation can be represented as follows:-

Covalent bond + Thermal energy ($> E_g$) $\rightarrow$ Electron + hole
(Thermal generation)

Covalent bond + optical illumination (Energy $> E_g$) $\rightarrow$ Electron + hole
(optical generation)

During process of EHP generation the conc. of electrons produced in CB is always equal to conc. of holes produced in V.B. i.e. $n = p$

(for EHP generation
for any type of semiconductor
intrinsic or n-type or p-type)

Once EHPs are generated optically or thermally, these move independently. The electrons produced move in CB and holes produced move in VB. In the absence of any external battery/voltage, the electrons in conduction band and holes in valence band move randomly, so that net current through semiconductor is

zero. If E_{TH} and $h\nu$ are thermal ~~and~~ energy and optical energy per absorption or ^{for} EHP generation then we must have

$$E_{TH} \geq E_g \quad \text{and} \quad h\nu \geq E_g$$

Now we can use same argument for generation of majority carrier for intrinsic semiconductor. That is thermal or optical ~~and~~ energy can produce a majority carrier from dopant atom. For example for n-type semiconductor (see page no. 23) a majority carrier (electron) can be generated by giving energy ΔE from donor level to conduction band. i.e.



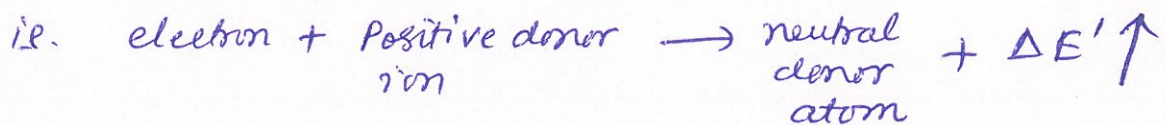
$$\text{where } \Delta E \geq (E_C - E_D)$$

However in such carrier generation process, no holes are created in ~~the~~ valence band and instead positive ions are created in donor level. Similarly reader can explain generation of holes (majority carriers) in p-type semiconductors due to transition of electron from ~~donor~~ VB to acceptor level.

Recombination: - ~~The~~ The electrons produced in CB lose their energy due to collision with other particles in lattice and eventually fall back to VB, where they combine with holes. This process is called carrier recombination and it leads to formation of covalent bond and release of energy $\Delta E (\geq E_g)$ for an intrinsic semiconductor. The process of carrier recombination for intrinsic semiconductor can be represented as follows:-



Not all the electrons fall to V.B. during recombination when material is extrinsic semiconductor. For example for n-type semiconductor electron leaving C.B. may combine with positively charged donor ions and in the process small amount of energy $\Delta E' (\approx E_c - E_D)$ is released.



~~Reader~~

Note that during this kind of recombination process, no covalent bond is formed, ~~and~~ instead a neutral impurity atom (donor for n-type & acceptor for p-type) is formed and energy released $\Delta E' (\approx E_c - E_D)$ is very small.

(Reader can himself/herself create similar explanation for recombination process b/w E_A and V.B. for p-type semiconductor).

The process of recombination is very important. When external thermal energy or optical illumination provided to semiconductor is switched off, then process of carrier recombination starts dominating ^{and} due to excess electron-hole pair recombination, material returns back to or achieves back state of thermal equilibrium. It should be noted that optically generated EHPs are responsible for the working of light dependent resistors (LDR), photodiodes and ^{other} optical detector devices etc.

Drift and diffusion currents:- We know that electrons and holes move randomly in a semiconductor crystal in the absence of any external electric field. Due to this reason, the average velocity of charge carrier is zero and no current is flowing in the semiconductor. However, it is possible to disturb this thermal equilibrium, so that most of carriers move in ~~an~~ a unique direction, so that a current may start flowing in semiconductor. Depending on the manner in which this thermal equilibrium can be broken in a semiconductor, the current that can flow through it is classified in two categories. That is (i) Drift current (ii) Diffusion current.

1. Drift Current:- When an external electric field E is applied across a semiconductor, then positive and negative charge carriers experience a force ~~over and above the~~ which is given by the expression:

$$\left. \begin{aligned} \vec{F}_e &= -e\vec{E} \\ \text{and } \vec{F}_h &= +e\vec{E} \end{aligned} \right\} \text{--- ①}$$

That is force on holes is in the direction of electric field and force on electrons is opposite to the direction of electric field. Due to this ^{electric} force, electrons and holes acquire a directional motion over and above their random (thermal) motion. "The current produced due to motion of charge carriers under the effect of external electric field is called drift current."

Let $\vec{E}$ = applied electric field

μ_e = mobility of electron

μ_h = " " hole

e = magnitude of charge on electron/hole

J_{ed} = magnitude of electron current density due to drift

J_{hd} = magnitude of hole current density due to drift

$\vec{v}_e$ = drift velocity of electron (opposite to $\vec{E}$)

$\vec{v}_h$ = drift velocity of hole (in the direction of $\vec{E}$)

n = electron concentration

p = hole concentration

$$\therefore J_{ed} = ne\mu_e E$$

$$\therefore J_{hd} = p e \mu_h E$$

Although motion of electron & hole is in opposite direction, but convention current of both carriers is in the same direction (shown by dotted arrows in fig. 1.). Thus total drift current density in a semiconductor is equal to the sum of drift current densities of electron & hole. i.e.

$$J_d = J_{ed} + J_{hd} \quad (\text{where } J_d = \text{total drift current density})$$

$$= ne\mu_e E + p e \mu_h E$$

$$\therefore \boxed{J_d = e(n\mu_e + p\mu_h)E}$$

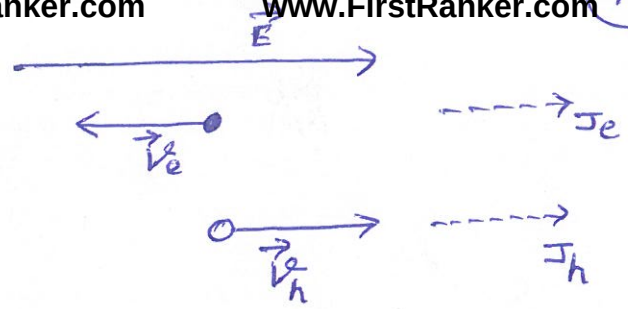


Figure (1)

2. Diffusion current :- In case of semiconductors, we can produce directional motion of charge carriers (in addition to their random thermal motion) not by applying external electric field but also by creating difference in conc. of charge carriers (called concentration gradient) along the length of semiconductor. The current created in this fashion is called "Diffusion current."

"Diffusion is the process of movement of one kind of particles from region of high conc. to region of low conc."

Thus existence of conc. gradient is the necessary condition for diffusion process to take place.

The process of diffusion continues till conc. becomes ~~the~~ uniform through a system.

In case of semiconductors, since the diffusing particles (electrons/holes) are charged, therefore this diffusion creates current in the semiconductor, which is called diffusion current.

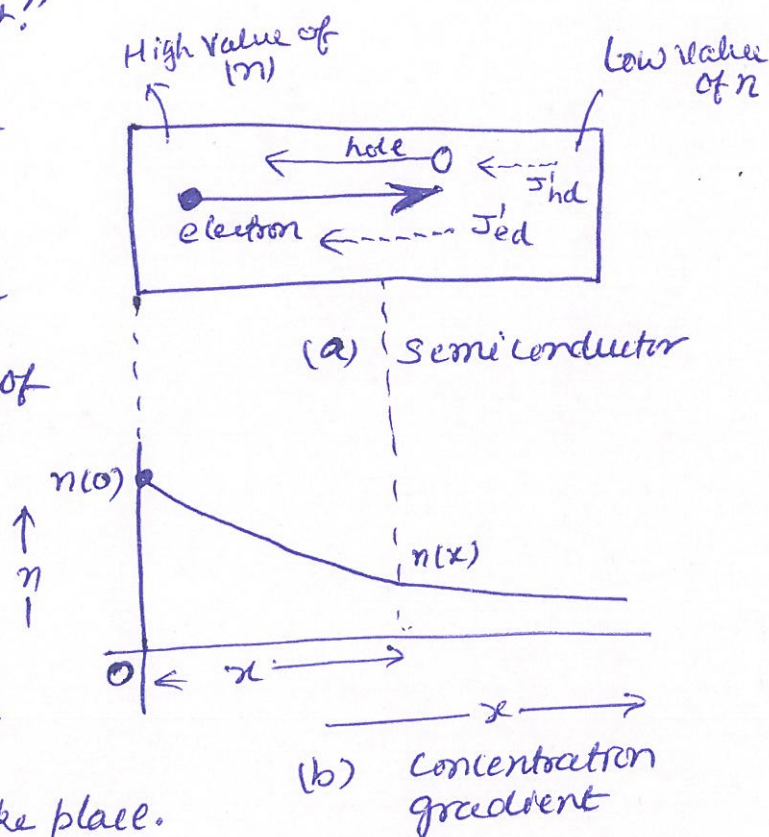


Fig. (2)

Conc. gradient may be produced in semiconductors, by applying heat or light locally at one end of the semiconductor sample. This generates additional EHPs. Due to this, there is a sudden change in conc. of electrons and holes across length of semiconductor and diffusion currents are set up in the ~~sem~~ sample.

Let $\frac{dn}{dx}$ = change in conc. of electrons along small distance dx (called conc. gradient of electrons)

Let $\frac{dp}{dx}$ = conc. gradient of holes

~~For the~~ For any situation $\frac{dn}{dx}$ & $\frac{dp}{dx}$ are always of opposite signs ($\because$ If electron conc. is more at one end, then hole conc. will be more on opposite end). As per situation described in fig. (2) above, value of n is decreasing along increasing value of x . Therefore, $\frac{dn}{dx}$ is negative and $\frac{dp}{dx}$ will be positive.

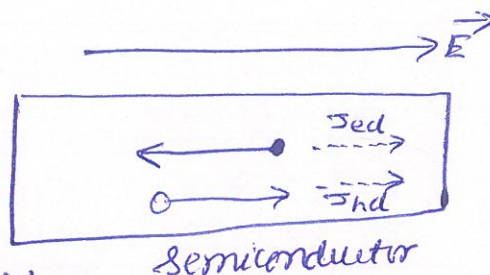
Let J'_{ed} = electron diffusion current density

J'_{hd} = hole " " "

It is observed that diffusion current density is directly proportional to conc. gradient of charge carriers and charge on the diffusing particle.

i.e. $J'_{ed} \propto (+e)$
 $\propto \left(\frac{dn}{dx}\right)$

$$\Rightarrow J'_{ed} = -e D_e \frac{dn}{dx} \quad \text{--- (1)}$$



Here D_e is called diffusion coefficient for electron motion. Equation (1) gives magnitude of diffusion current density of electrons. Negative sign in (1) is due to the fact that $\frac{dn}{dx}$ is negative, so that J'_{ed} remains positive.

Similarly diffusion current density for holes is given as

$$J'_{hd} = e D_h \frac{dp}{dx} \quad \text{--- (2)}$$

Where D_h is diffusion coefficient for hole motion.

In a semiconductor drift and diffusion currents co-exist. Thus total current density is the resultant of both current densities. Let electric field $\vec{E}$ is applied to semiconductor as shown in fig (2). Then comparison of Fig (1) & (2) shows that net electron current density is given as:-

$$J_e = J_{ed} - J'_{ed} \quad (\because \text{both are in opposite direction})$$

$$= n e \mu_e E - (-e D_e \frac{dn}{dx})$$

$$\boxed{J_e = e \left(n \mu_e E + D_e \frac{dn}{dx} \right)}$$

Similarly net hole current density is given as

$$J_h = J_{hd} - J'_{hd} \quad (\because \text{both are in opposite direction})$$

$$= p e \mu_h E - e D_h \frac{dp}{dx}$$

$$\Rightarrow \boxed{J_h = e \left(p \mu_h E - D_h \frac{dp}{dx} \right)}$$

SC

Carrier Transport:- Any process, that is responsible for the movement of charge carriers within a semiconductor is called carrier transport. There are the possible mechanisms that lead to carrier transport within the semiconductors:

(i) Thermal motion (ii) carrier drift (iii) carrier diffusion

(i) Thermal motion:- At any given temperature above 0K, carriers have thermal kinetic energy $\frac{1}{2}kT$ per degree of freedom. This thermal energy gives the tendency to charge carriers to move within the crystal.

However during their motion, these carriers collide with vibrating lattice ions due to which their direction of motion changes after every collision



Fig 1.
(Brownian motion)

As shown in Fig.1. Such kind of motion is called ~~thermal motion~~ Brownian motion.

The thermal energy can also lead to interactions of charge carriers with positively or negatively charged dopant atoms or among the interactions between the charge carriers themselves. Both of these processes (i.e. collision with semiconductor lattice or interactions of carriers within themselves or with impurity ions) are responsible for random (or Brownian) motion of charge carriers.

Let $\tau_c = \frac{\text{average}}{\text{time between two successive collisions}}$

SC

τ_c is also called relaxation time

Let v_{th} = thermal velocity of a charge carrier

λ ~~is~~ mean free path (i.e. average distance travelled between two successive collisions)

$$\therefore v_{th} = \frac{\lambda}{\tau_c} \quad \text{or} \quad \boxed{\lambda = \tau_c v_{th}}$$

Typical values of τ_c and v_{th} are

$$\tau_c \approx (10^{-14} - 10^{-13}) \text{ s}$$

$$v_{th} \approx 10^7 \text{ cm/s}$$

$$\text{so that } \lambda \approx (1-10) \text{ nm}$$

It should be noted that due to random motion, the average thermal velocity of charge carriers is zero.

(ii) Carrier drift :- The motion of charge carriers ^{due to} ~~under~~ external electric field is called carrier drift. The ~~exp~~ expressions for drift currents (see previous article) are given as

$$J_{ed} = n e \mu_e E$$

$$\text{and } J_{hd} = p e \mu_h E$$

(iii) ^{carrier} Diffusion :- The motion of charge carriers ~~in~~ due to conc. gradient in a semiconductor is called carrier diffusion. The expressions for diffusion currents (see previous article) are given as

$$J'_{ed} = - e D_e \frac{dn}{dx}$$

$$\& J'_{hd} = e D_h \frac{dp}{dx}$$

SC

Since average thermal velocity of charge carriers is zero. Therefore, although there are three mechanisms responsible for carrier transport, but only two of these (viz., drift and diffusion) are responsible for generation of current within a semiconductor. Thus expressions for total electron & hole current densities (See last article) are given by

$$J_e = J_{ed} + J_{ed}'$$

$$= e \left(n \mu_e E + D_e \frac{dn}{dx} \right)$$

$$\text{and } J_h = J_{hd} + J_{hd}'$$

$$= e \left(p \mu_h E - D_h \frac{dp}{dx} \right) = e \left(p \mu_h E - D_h \frac{dp}{dx} \right)$$

If J = total current density, then

$$J = J_e + J_h \quad (\text{since both } J_e \text{ \& } J_h \text{ are in same direction})$$

$$\Rightarrow J = e \left[n \mu_e + p \mu_h \right] E + e \left(D_e \frac{dn}{dx} - D_h \frac{dp}{dx} \right)$$

$$\Rightarrow J = e \left[(n \mu_e + p \mu_h) E + D_e \frac{dn}{dx} - D_h \frac{dp}{dx} \right]$$

(iv) Einstein's relations : Although diffusion and drift appear to be two different processes (because their origin is not the same, one is due to conc. gradient & other due to electric field), the parameters like mobility (μ_e, μ_h) and

SC

diffusion coefficient (D_e, D_h) are ~~not same~~ not independent.

There exists a close relation between them, since both of these parameters are determined by the thermal motion and scattering of the free carriers. The relation between these carriers are given below:-

$$\frac{D_h}{\mu_h} = \frac{kT}{e} \quad \text{--- (a)}$$

$$\frac{D_e}{\mu_e} = \frac{kT}{e} \quad \text{--- (b)}$$

~~If we divide (a) by~~

Equations (a), (b) are called Einstein relations. We can also equate (a) & (b) to write

$$\boxed{\frac{D_h}{\mu_h} = \frac{D_e}{\mu_e}}$$

Light Emitting diode (LED):- It is a semiconductor device, that produces radiation in the visible region, when it is forward biased. LEDs are generally fabricated using group III-IV compound semiconductors as base material (like ~~GaAs~~ Gallium - Arsenide or GaAs) and a suitable impurity (like Phosphorus), so that energy gap between V.B. and C.B. (i.e. E_g) falls in the visible region.

SC

Principle:- When a p-n junction diode (compound semiconductor) is forward biased, then large number of majority carriers cross the junction (due to conc. gradient i.e. diffusion process) and combine with minority carriers. Similarly minority carriers also cross the junction (due to applied electric field i.e. drift current) and recombine with majority carriers. During the recombination, new covalent bonds are formed and kinetic energy of charge carriers is emitted in the form of electromagnetic radiation. This process is called "injection electroluminescence."

Let E_g = band gap energy

ν = frequency of emitted photon

λ = wavelength " " "

$$E = h\nu = \frac{hc}{\lambda} = \text{energy of emitted photon}$$

$\therefore$

$$E \geq E_g$$

$$\text{or } E_{\min} = E_g$$

$$\Rightarrow h\nu_{\max} = E_g$$

$$\text{or } \frac{hc}{\lambda_{\min}} = E_g$$

$$\Rightarrow \boxed{\nu_{\min} = \frac{E_g}{h}} \quad \text{--- (1)}$$

$$\therefore \boxed{\lambda_{\min} = \frac{hc}{E_g}} \quad \text{--- (2)}$$

If energy E_g is expressed in units of eV and wavelength $\lambda_{\min}$ is expressed in units of μm , then relation (2)

SC

can be expressed as

$$\lambda_{min} = \frac{1.24 \mu m}{E_g (eV)}$$

A significant light output requires large value of recombination rate. It means that compound semiconductor must be heavily doped.

Theory: ~~working~~

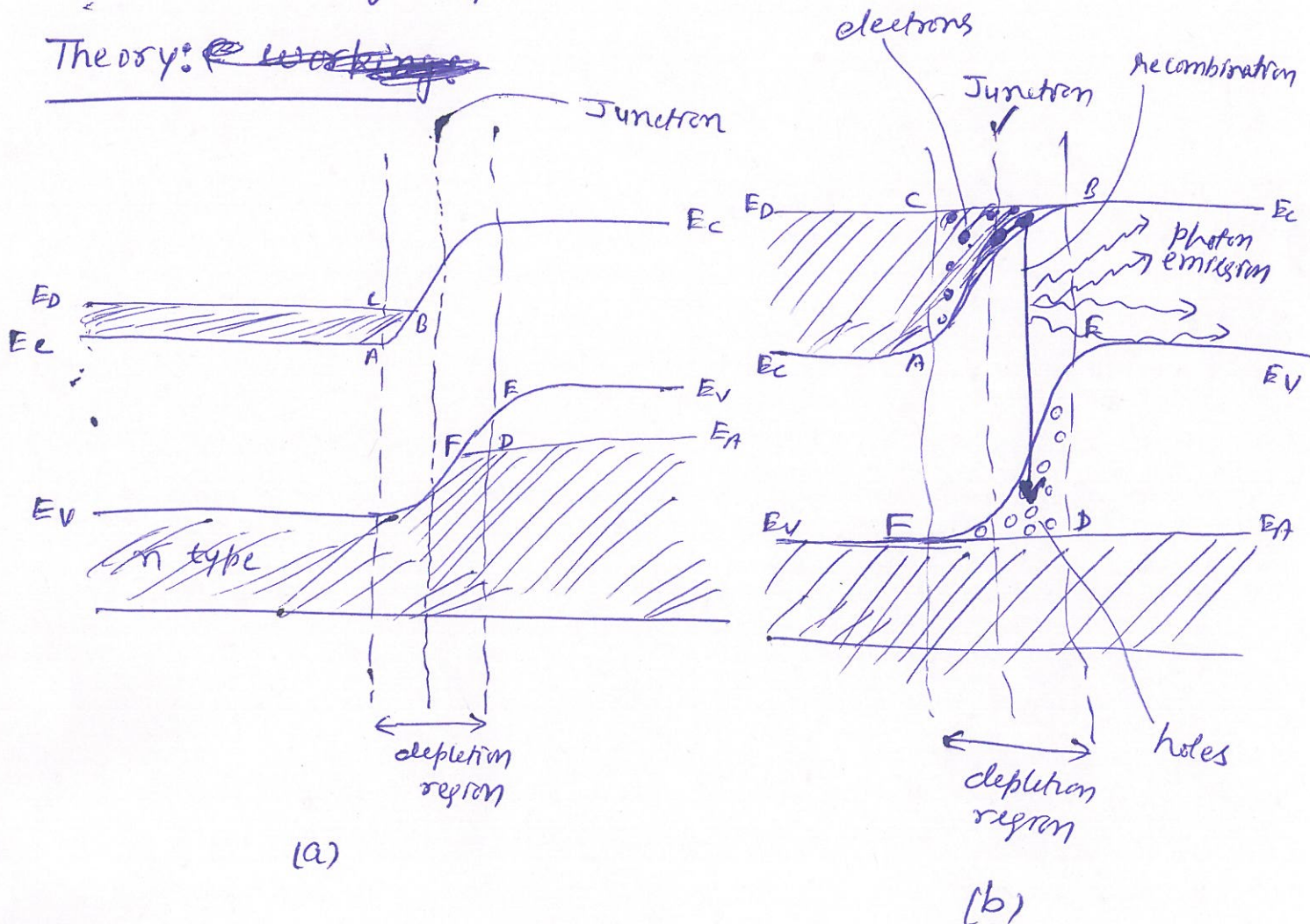


Figure 1.

The energy band diagram of heavily doped compound semiconductor diode without any forward bias (no connections to battery) is shown in part (a) & with forward bias is shown in part (b) of Figure 1. above.

SC

Here regions ABC and DEF are part of donor and acceptor bands respectively. In the absence of any forward bias, donor band region ABC lies in n-side only and acceptor band region DEF lies in p-side only. However when forward bias is applied (fig 1(b)), then due to supply of electrons to ^{n side of} Semiconductor by battery ~~from~~ results in widening of donor band region ABC. Similarly due to withdrawal of electrons by battery from p-side, the ~~at~~ empty acceptor band region DEF also widens up, so that both of these regions (ABC & DEF) ~~extend~~ cover whole depletion region and ABC (filled region ^{containing electrons}) is directly above DEF (empty region containing holes). Due to this, recombination process between electrons and holes start taking place and photons are emitted in the process due to loss of kinetic energies during formation of covalent bonds during recombination.

~~The amount of energy released is~~

The energy (or wavelength) of emitted photon depends on the band gap energy E_g . This band gap energy E_g is itself a function of nature and conc. of doping in the compound semiconductor used for making LED. The wave length range (or colour) of emitted

Light for different amount of doping is given in table below:-

Material used	Colour range of emitted light
Gallium-Arsenide (GaAs)	IR region
$GaAs_xP_{1-x}$	Red or yellow (For suitable values of x $0 \leq x \leq 1$)
GaP_x	Red or Green (for suitable values of x) $0 \leq x \leq 1$

Construction & Working:-

Schematic diagram of a surface emitting LED is given in fig. 2. An n-type substrate is taken and a very thin layer of p-region is produced at its surface using process of diffusion of acceptor atoms

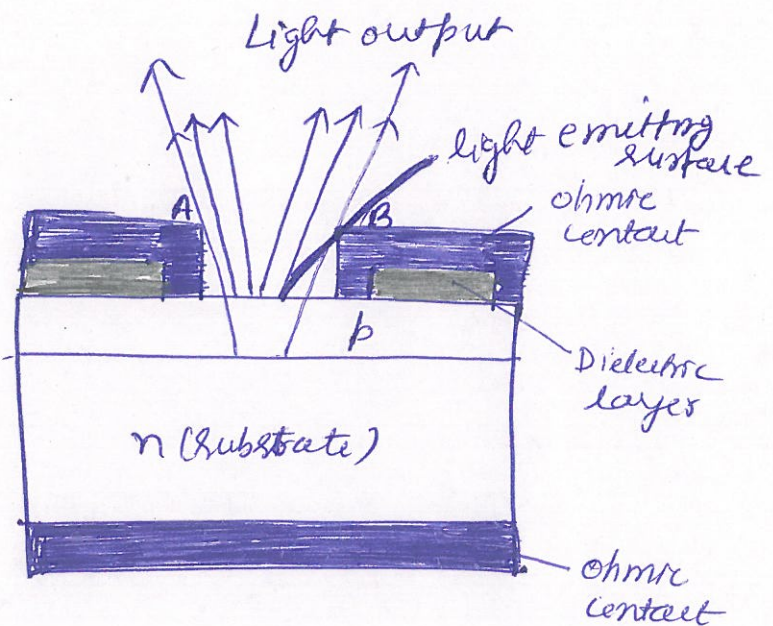


Fig. (2)

In n-type substrate. Thin layer of p-type material ensures that junction and hence depletion region (which is responsible for emission of light as recombination is dominating in this region only) is very close to the

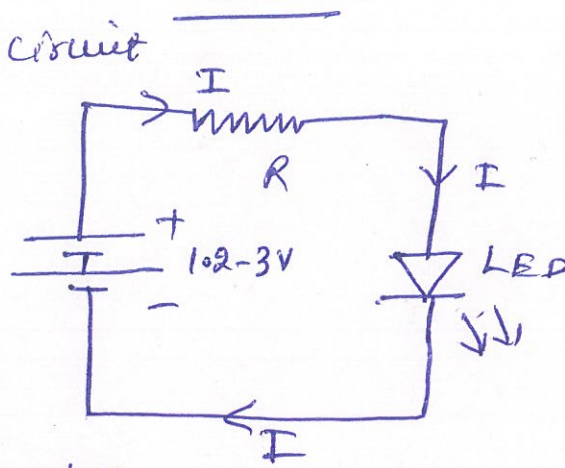
light emitting surface. This ensures that light photons produced during recombination process after applying forward bias are not absorbed by the material during their travel process from depletion region to the light emitting surface.

There is another issue, which needs to be addressed at light emitting surface, that is of total internal reflection (TIR), which can take place as light is coming out of ~~denser~~ p-region (denser) into air (rarer). To avoid this, a thin transparent dielectric layer (epoxy resin) of suitable refractive index (slightly more than refractive index of p-region) is deposited over light emitting surface.

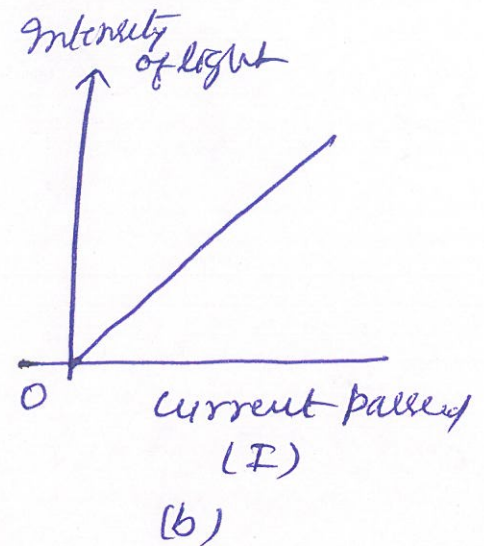
~~In order to get light~~

The ~~pi~~ both surfaces of LED diode are having metal contacts and there is a small gap AB on this metal contact on the light emitting surface through which light can come out. These metal/ohmic contacts serve two purposes. (i) These help in ~~connect~~ establishing the external forward bias connections with battery (ii) The light photons produced during recombination may travel in any direction and these photons are reflected by these metal contacts (which act as mirrors from inner surface) Reflect the photons so that they come out of only ~~rear~~ region AB and output intensity gets

increased. The circuit diagram of LED in forward bias is shown in Fig 3(a).



Here resistance R is used to control current passing (a)

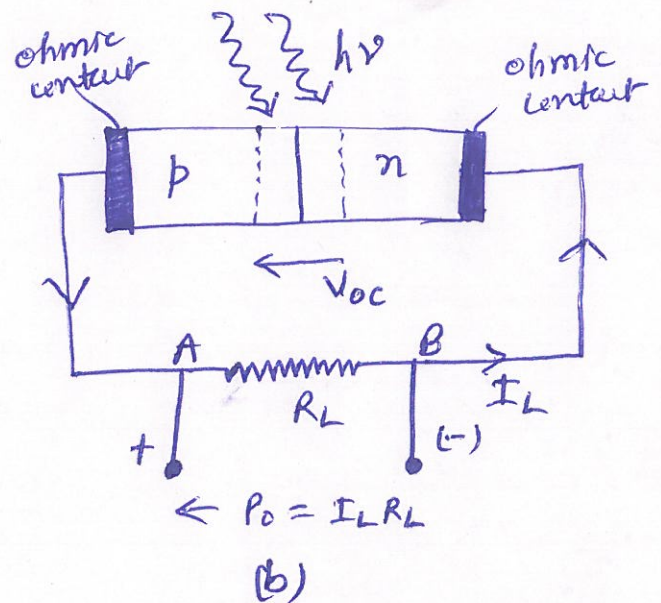
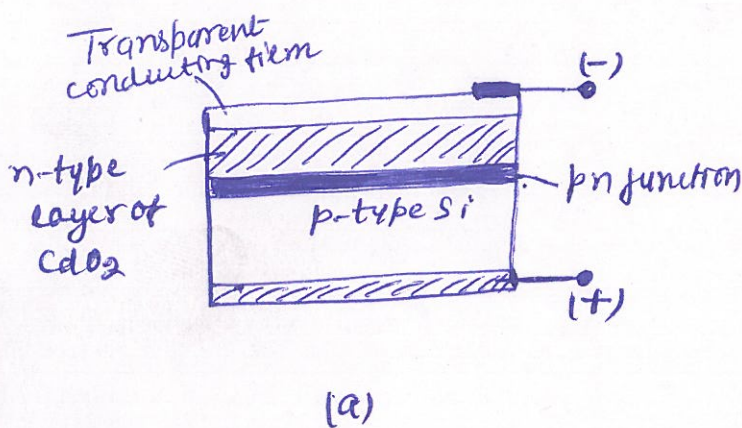


through LED in forward bias, Fig 3.

because typically a LED ~~gets burnt~~ has a burning current of 20mA. The resistance R ensures that current passing through diode is always less than this value. The forward bias voltage for LED is between 1.2 to 3V (approx.) depending upon material of device. It is observed that LED is a current controlled device i.e. intensity of emitted light is directly proportional to forward bias current (I) as shown in Fig 3(b).

The reverse breakdown voltage of LED is very small (typically 3V) as compared to normal p-n junction diode (typically 25V). Since forward operating voltage of LED (1.2-3V) is also comparable with reverse reverse breakdown voltage, so it should be kept in mind that LED should never be reverse ~~be~~ biased, otherwise it will get permanently damaged at the same instant.

Solar cell :- A solar cell is a p-n junction device, that can deliver ~~off~~ electric power, when illuminated with sunlight or visible radiation from another light source. ~~These~~ Solar cells are usually connected in large number to form solar panels. This serves two purposes first the surface area becomes very large by making solar panels, due to which large amount of optical energy can be accumulated, secondly the emf generated due to ~~a single~~ ^{all cells} solar cells can be gets added to ~~go~~ develop appreciable voltage.



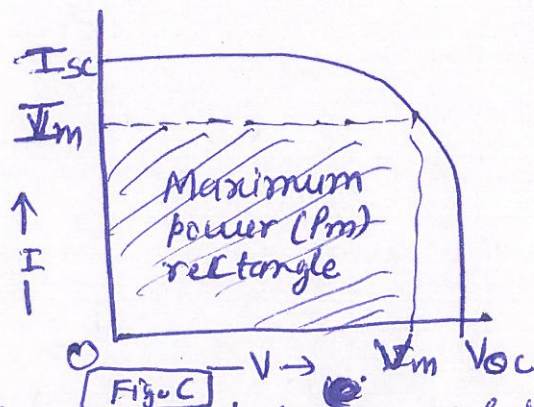
The schematic diagram of a solar cell is given in fig (a) above.

It consists of a p-type Si chip over which a thin layer of n-type material (usually CdO_2) is grown. When solar radiation is incident on the cell from n-side, EHPs are generated in n and p regions near the depletion region (or space charge region). The electric field of depletion (or space charge) region does not allow these generated carriers to combine.

Electrons generated in p-region are drawn into n-region and holes generated in n-region are drawn into p-region. It results in accumulation of charge on the two sides of junction and produces an emf, which is called Photo emf. The magnitude of emf is approximately 0.5V. The overall power conversion efficiency of a solar cell is ranges from 10%-30% and power output ranges from 10-30 mW/cm². It means if we desire to generate a power of 20W, then approximate value of area of solar panel will be $\left(\frac{10+30}{2}\right) \text{ mW/cm}^2 \times \text{Area} = 20W$
 $\Rightarrow 10^{-3} \times \text{Area} = 1$

$$\Rightarrow \text{Area} = \frac{1}{10^{-3}} = 1000 \text{ cm}^2$$

Due to accumulation of charges on the end faces of solar cell, a current will flow through the external circuit,



if a load resistance R_L is connected between n & p terminals as shown in figure b. In the external circuit, the current will flow through R_L from p-side to n-side of semiconductor i.e. end A of R_L is at high potential w.r.t. end B. The net power generated depends on the solar cell and value of load connected across it.

The I-V characteristic of a solar cell is shown in fig. (c) above. When no load resistance is

connected b/w terminals of solar cell, then emf developed is called open circuit voltage V_{oc} (at this voltage current I_L in the circuit is zero). On the other hand if we join two terminals of a solar cell with a wire of zero resistance (This is called short circuiting), then maximum current (that a solar cell can deliver) passes through external circuit. This current I_{sc} is called short circuit current. When we note V_{oc} & I_{sc} , these values are shown on the IV characteristic of solar cell. Now we can obtain value of current for different values of load resistance (R_L). It should be noted that V_{oc} is close to the turn-on voltage of solar cell, while I_{sc} is close to the photocurrent. The net power is equal to product of voltage across solar cell and current drawn from it. The value of power initially increases with increase in voltage and then rapidly goes to zero around turn on voltage of diode. The maximum power is obtained at a voltage labelled as V_m , corresponding to which current is I_m .

Solar cells can be connected in series or parallel in a solar cell and the power generated by solar panels can be of several kilowatts.

