

→ Why we need to study this concept?

Ans:- When we are finding electrical properties of semiconductors, we have to know that, the no. of charge carriers per cm^3 are available.

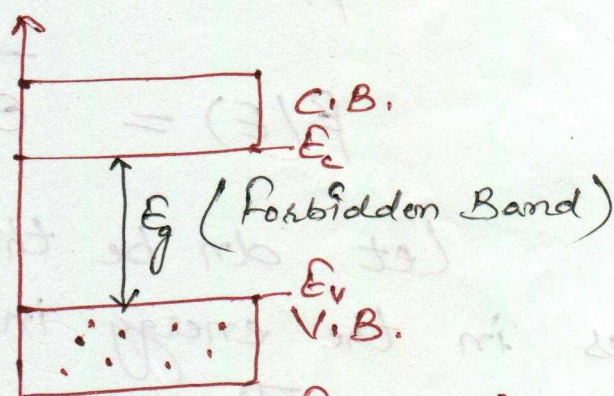
→ To analyze the semiconductor devices, we need to know how many number of charge carriers are available.

To study this concept, we need to know about

- (i) The Fermi-Dirac distribution function.
- (ii) The Fermi Level
- (iii) Electron and Hole concentrations at equilibrium.

Energy Band Diagram

To analyze the distribution of electrons over a range of allowed energy levels at



thermal equilibrium, we can use Fermi Dirac Function $f(E)$ and is given by

$$f(E) = \frac{1}{1 + e^{\frac{E - E_f}{KT}}}$$

(FD Distribution function)

E_f is called Fermi Level

The Probability Distribution function also gives the Probability of allowed energies occupied by the electrons. Hence, it is also called Probability Distribution Function.

Calculation of Electron Density :-

When the no. of electrons is very small compared to the available energy levels, the Probability of an energy state being occupied by more than one electron is very small. Such a situation is valid when $(E - E_F) \gg 3KT$. Under this circumstance, the no. of available states in the C.B. is far larger than the no. of e's in the Band. Then, the P.D. Function can be approximated to Boltzmann function,

$$P(E) = \frac{e^{-(E-E_F)/KT}}{e} \quad \text{--- (2)}$$

Let dn be the no. of e's whose energy lies in the energy interval E and $E+dE$ in the C.B. Then,

$$dn = Z(E) P(E) dE \quad \text{--- (3)}$$

Where $Z(E)dE$ is the density of states in the Energy Interval E and $E+dE$ and $P(E)$ is the Probability that a state of energy is occupied by an electron.

Thus, the electron density in the C.B. can be found by integrating eqⁿ (3) between the limits E_c and ∞ . E_c is the energy corresponding to the bottom edge of the C.B. and ∞ , the energy corresponding to the top edge of the C.B.

$$\therefore n = \int_{E_c}^{\infty} Z(E) F(E) dE \quad \text{--- (4)}$$

The density of states in the C.B. is given by

$$Z(E) dE = \frac{4\pi}{h^3} (2m_e)^{3/2} (E - E_c)^{1/2} dE \quad \text{--- (5)}$$

Here $(E - E_c)$ be the kinetic energy of the conduction $\bar{e}$ at higher energy levels.

Thus, putting values from eqⁿs (2) and (5) in eqⁿ (4), we obtain

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3} (2m_e)^{3/2} (E - E_c)^{1/2} \cdot e^{-(E - E_F)/KT} dE$$

or

$$\Rightarrow n = \frac{4\pi}{h^3} (2m_e)^{3/2} e^{(E_F - E_c)/KT} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-(E - E_c)/KT} dE \quad \text{--- (6)}$$

The integral in eqⁿ (6) is of the standard form of type

$$\int_0^{\infty} x^{1/2} e^{-ax} dx = \frac{\sqrt{\pi}}{a\sqrt{a}} \quad \text{Where } a = \frac{1}{KT} \text{ and } x = (E - E_c)$$

$$\therefore n = \frac{4\pi}{h^3} (2m_e)^{3/2} e^{-(E_F - E_c)/KT} \left[\frac{\sqrt{\pi}}{2} (KT)^{3/2} \right]^4$$

Rearranging the terms, we get

$$n = 2 \left[\frac{2\pi m_e KT}{h^2} \right]^{3/2} e^{-(E_c - E_F)/KT} \quad (4)$$

This is the expression for electron concentration in the C.B. of an Intrinsic Semiconductor.

$$\Rightarrow n = N_c e^{-(E_c - E_F)/KT} \quad (5)$$

$$N_c = 2 \left[\frac{2\pi m_e KT}{h^2} \right]^{3/2}$$

where N_c is a

temperature dependent material constant known as the effective density of states in the C.B.

Eqn (4) shows the dependence of Fermi-

level on electron concentration.

Calculation of Hole density :-

In this case $(E_v - E)$ will be the kinetic energy of the hole at lower energy levels. So, the F.D function can be approximated as

$$F(E) = e^{-\frac{(E_F - E)}{KT}} \quad (1)$$

and

$$P = \int_{-\infty}^{E_v} Z(E) F(E) dE \quad (2)$$

Here, $Z(E) dE = \frac{4\pi}{h^3} (2m_h)^{3/2} (E_v - E)^{1/2} dE \quad (3)$

Put (1) and (3) in eqⁿ (2)

$$\begin{aligned} P &= \frac{4\pi}{h^3} (2m_h)^{3/2} \int_{-\infty}^{E_v} (E_v - E)^{1/2} e^{-\frac{(E_F - E)}{KT}} dE \\ &= \frac{4\pi}{h^3} (2m_h)^{3/2} e^{-\frac{(E_F - E_v)}{KT}} \int_{-\infty}^{E_v} (E_v - E)^{1/2} e^{-\frac{(E_v - E)}{KT}} dE \end{aligned}$$

$$P = \frac{4\pi}{h^3} (2m_h)^{3/2} e^{-\frac{(E_F - E_v)}{KT}} \left[\frac{\sqrt{\pi}}{2} (KT)^{3/2} \right]$$

Rearranging the terms, we get

$$P = 2 \left[\frac{2\pi m_h KT}{h^2} \right]^{3/2} e^{-\frac{(E_F - E_v)}{KT}} \quad (4)$$

The above eqⁿ is the expression for the Hole Concentration in the V.B of an Intrinsic Semiconductor.

⇒

$$P = N_v e^{-\frac{(E_c - E_v)}{KT}}$$

Where N_v is called the effective density of states in the V.B.

Eqⁿ (4) shows the dependence of fermi-level on Hole concentration.

Intrinsic Density or Intrinsic Concentration

A single event of Bond Breaking in a pure semiconductor leads to the generation of an electron-hole pair. At any temp. T , the no. of e^- generated will be equal to the no. of holes generated. As, the two charge carrier concentrations are equal, they are denoted by a common symbol n_i , which is called Intrinsic Density or Intrinsic Concentration. Thus,

$$n = p = n_i$$

$$\Rightarrow n_i^2 = np = (N_c N_v) e^{-\frac{(E_c - E_v)}{KT}}$$

$$\Rightarrow n_i^2 = (N_c N_v) e^{-\frac{E_g}{KT}} \quad [\because E_c - E_v = E_g]$$

On Putting the values of N_c and N_v from above we obtain

$$n_i = 2 \left[\frac{2\pi KT}{h^2} \right]^{3/2} (m_e m_h)^{3/4} e^{-\frac{E_g}{4KT}}$$

This is the expression for Intrinsic Carrier Concentration.

Variation of Intrinsic Carrier Concentration with Temperature

The above eqⁿ (A) can be written as

$$n_i = 2 \left[\frac{2\pi k}{h^2} \right]^{3/2} (m_e m_h)^{3/4} \frac{1}{T} e^{-E_g/2kT} \quad \text{--- (B)}$$

$$\Rightarrow \boxed{n_i \propto T^{3/2}}$$

From eqⁿ (B), the following points can be analyzed:

- (i) The intrinsic concentration is independent of Fermi level.
- (ii) The Intrinsic concentration has an exponential dependence on the Band Gap value E_g .
- (iii) It strongly depends on temperature.
- (iv) The factor 2 in the exponent indicates that two charge carriers are produced for one covalent bond broken.

For Numericals $\Rightarrow$ The Intrinsic Charge Carrier concentration may be approximated to

$$\boxed{n_i = 10^{21.7} \frac{1}{T^{3/2}} \times 10^{-2500 E_g / T}}$$

Note :- $n \cdot p = n_i^2$ is also called as

Mass-Action Law.

* The probability that an electron being thermally promoted to the C.B. is given by

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} = \frac{1}{1 + e^{E_g/2kT}}$$

$$\left[\because E_c - E_F = \frac{E_g}{2} \right]$$

* Fraction of e^- in the C.B. : $e^{-E_g/2kT}$

$$\frac{n}{N} = \frac{1}{\exp\left[\frac{E_g}{2kT}\right]} = e^{-E_g/2kT}$$

$n \rightarrow$ no. of e^- excited to C.B. levels.

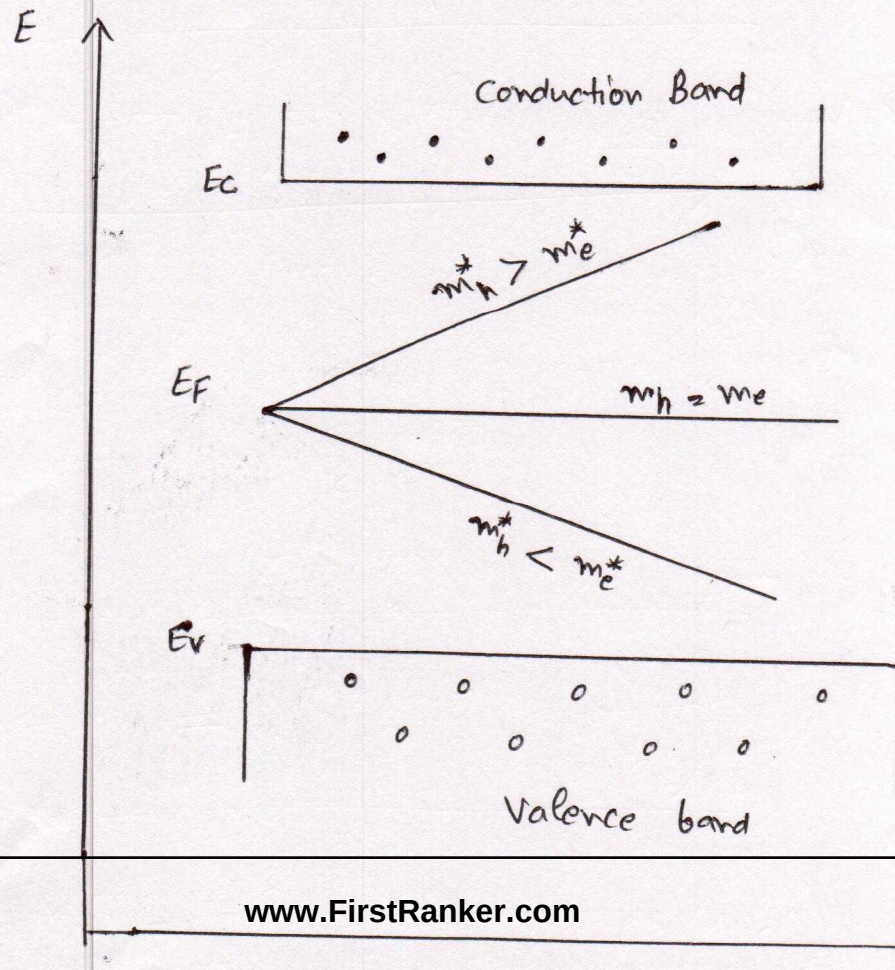
$N \rightarrow$ total no. of e^- available in the V.B. initially.

Note :- These formulas are important for Numerical Problems.

Variation of Fermi Level with Temperature in an Intrinsic Semiconductor

With an increase in temperature, the fermi level gets displaced upward to the bottom edge of the C.B. if $m_h > m_e$ or downward to the top edge of the V.B. if $m_h < m_e$ (as shown in fig.)

In most of the materials, the shift of Fermi level on account of $m_h \neq m_e$ is insignificant. The Fermi Level in an Intrinsic Semiconductor may be considered as staying in the middle of the Band gap.



We know that the thermal equilibrium concentration of electrons and holes are:

$$n = N_c e^{-(E_c - E_F)/KT} \text{ and } \text{--- (1)}$$

$$p = N_v e^{-(E_F - E_v)/KT} \text{ --- (2)}$$

Where E_F - Intrinsic Fermi Energy

But we know that at thermal equilibrium for intrinsic semiconductor, we have

$$\Rightarrow n = p$$

$$\Rightarrow N_c e^{-(E_c - E_F)/KT} = N_v e^{-(E_F - E_v)/KT}$$

Taking Natural Logarithm on both sides, we get

$$\ln N_c + \left[\frac{-(E_c - E_F)}{KT} \right] = \ln N_v + \left[\frac{-(E_F - E_v)}{KT} \right]$$

$$KT \ln N_c - E_c + E_F = KT \ln N_v - E_F + E_v$$

$$\Rightarrow 2E_F = (E_c + E_v) + KT \ln \frac{N_v}{N_c}$$

$$\Rightarrow E_F = \left(\frac{E_c + E_v}{2} \right) + \frac{1}{2} KT \ln \frac{N_v}{N_c}$$

$$\Rightarrow E_F = E_{\text{MidBand Energy}} + \frac{1}{2} KT \ln \left(\frac{m_p}{m_e} \right)^{3/2}$$

(By Putting N_v & N_c values)

$$\Rightarrow E_F = E_{\text{MidBand Energy}} + \frac{3}{4} KT \ln \left(\frac{m_p}{m_e} \right) \text{ --- (3)}$$

From eqⁿ (3) following things are obtained :

- (1) If $m_h = m_e$ then $E_F = E_{midgap}$. Thus, the fermi level is exactly at the center of the band gap.
- (2) If $m_h > m_e$ then Fermi Energy level E_F is slightly above the center of the band gap.
- (3) If $m_h < m_e$ then E_F is slightly below the center of the band gap.

Extrinsic Semiconductor

n-type Semiconductor

p-type Semiconductor

1. Intrinsic + Pentavalent = n-type

Intrinsic + trivalent
= p-type

2. Electrons are majority charge carrier
(Holes are in minority)

Electrons are minority charge carrier
(Holes are in majority)

3. $n > p$

$p > n$

4. $n > n_i$ and $p < n_i$

$p > n_i$ and $n < n_i$

5.

We know that $n = N_c e^{-\frac{(E_c - E_F)}{KT}}$ — (1)

Taking logarithm on Both the sides, we get

$$\ln n = \ln N_c + \left[\frac{-(E_c - E_F)}{KT} \right]$$

$$\Rightarrow \frac{E_c - E_F}{KT} = \ln N_c - \ln(n)$$

$$\Rightarrow E_c - E_F = KT \ln \left(\frac{N_c}{n} \right) \text{ — (2)}$$

In n-type semiconductor, we have

$$N_d > n_i \text{ and } n \approx N_d$$

N_d - Effective density of states of Donor components

Thus,

$$E_c - E_F = KT \ln \left(\frac{N_c}{N_d} \right) \text{ — (3)}$$

Eqⁿ (3) represents the information regarding E_F for n-type semiconductor w.r.t. E_c . It is clear that E_F lies below E_c .

Similarly, in p-type semiconductor

$$E_F - E_v = KT \ln \left(\frac{N_v}{N_a} \right) \text{ — (4)}$$

Eqⁿ (4) represents the information regarding E_F for p-type semiconductor w.r.t. E_v . It is clear that E_F lies above E_v .

Variation of E_F with Doping Concentration

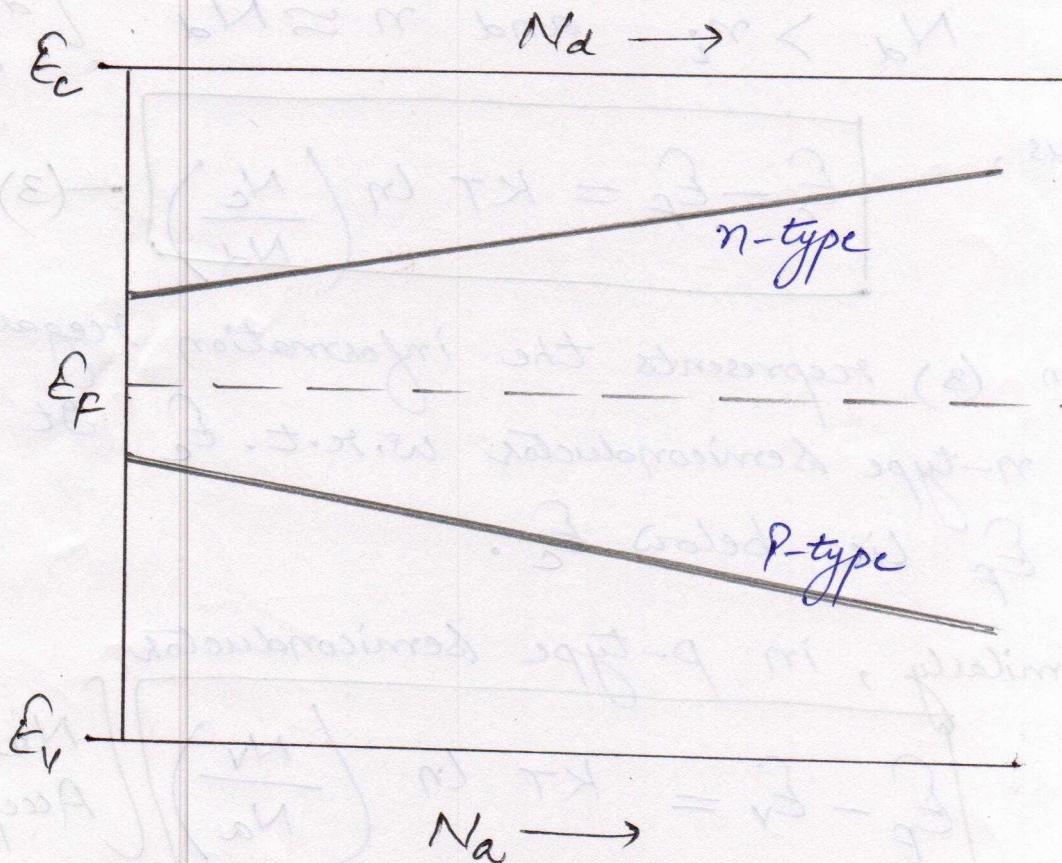
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We know that

$$E_c - E_F = KT \ln \left(\frac{N_c}{N_d} \right)$$

As N_d increases, $(E_c - E_F)$ decreases. Thus, E_F shifts towards the Conduction Band for n-type Semiconductor.

Similarly, E_F shifts towards the Valence Band for p-type Semiconductor as acceptor impurity concentration increases.



In general as Doping level increases the Fermi level shifts towards the Band

Position of E_F with Temperature

We know that, for n-type semiconductor the position of the Fermi level is given by

$$E_c - E_F = KT \ln \left(\frac{N_c}{N_d} \right). \quad \text{Thus,}$$

As the temperature increases, N_c increases and hence $(E_c - E_F)$ is also increases. Thus, E_F moves away from the Conduction Band.

Similarly for p-type semiconductor, we have

$$E_F - E_v = KT \ln \left(\frac{N_v}{N_a} \right). \quad \text{Thus, as}$$

the temperature increases, N_v increases and hence $(E_F - E_v)$ also increases. Thus, E_F moves away from the Valence Band.

Note $\Rightarrow$ At higher temperature, the semiconductor material loses its extrinsic characteristics and begins to behave more like an Intrinsic Semiconductor. Because on increasing the temperature thermal generation will occur that leads to the same concentrations of both the carriers i.e. the material becomes Intrinsic again. Thus, for a particular temperature value (critical temp. T_c), the semiconductor completely behaves as an intrinsic one and its conductivity increases with temperature.