Module - III
Oscillations and Waves

Lectures: 03

Pre Test Questions

1. (a) What is a periodic Phenomenon? Give few examples.

   (b) Define Oscillation. What is the difference between mechanical and non-mechanical Oscillation?

   (c) What is Phase? Define phase constant.

2. Define angular frequency and how it is related with time period. Give the relation. Give a physical relation showing the relation between them.

3. What is Simple Harmonic Motion? What is a simple pendulum? How can you relate the motion of a simple pendulum to Simple Harmonic Motion? Write the differential equation linking acceleration to displacement.

4. What are Waves? What is a pulse? What are the different types on waves?

5. What are Standing Waves? How do you define harmonics of standing waves?

6. What is Doppler's Effect?

7. Define Resonance. What do mean by Beats?

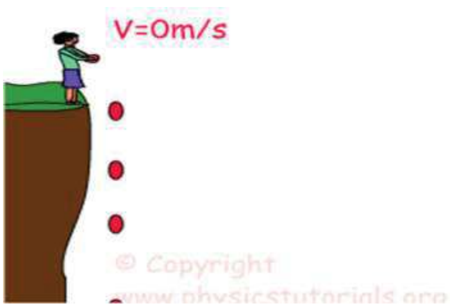
8. What is meant by Damping? Distinguish between under damping and over damping.

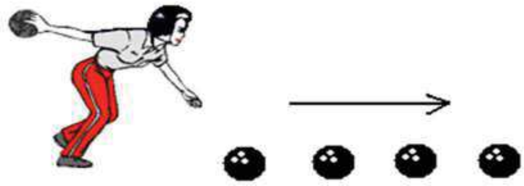
9. Distinguish between rarefaction and compression, and crest and trough.

10. How does a wave pulse interact for the following cases:
    i) When it encounters a fixed boundary?
    ii) When it encounters a free boundary?

Types of Motion: We very often come across the following types of motions

1. Rectilinear motion: Motion of a particle in a straight line





Examples: Motion of ball in straight line, a body that falls freely in vertical direction under the influence of gravity etc.

Periodic Motion

A motion that repeats itself at regular intervals of time is called **periodic motion**.

Swinging Pendulum and clock

In both the cases, the motions repeat after certain interval of time. Such a motion that repeats after certain interval of time is known as periodic motion. The body is displaced from a fixed point and it is given a small displacement, a force comes into action that tries to bring it to its equilibrium point, giving rise to oscillation or vibration.

Every oscillatory motion is a periodic, but every periodic motion need not be oscillatory.

Oscillations

- Oscillatory motion is a to and fro motion about a mean position and periodic motion repeats at regular intervals of time.
- All oscillatory motions are periodic but all periodic motions are not oscillatory.

Oscillations or **Vibrations**

There is no significant difference between oscillations and vibrations.

- Low frequency periodic motions are called as oscillation (like the oscillation of a branch of a tree),
- High frequency Periodic motions are called as vibration (like the vibration of a string of a musical instrument).

Periodic Motions which can be represented by Sinusoidal waveform (Sine or Cosine wave) are known as harmonic motion

Simple Harmonic Motion

- ***Simple Harmonic Motion (SHM)*** is a specialized form of periodic motion
- Periodic vibration around an equilibrium position
- Restoring force must be
    - proportional to displacement from equilibrium
    - in the direction of equilibrium

There are two types of SHM that will be discussed.

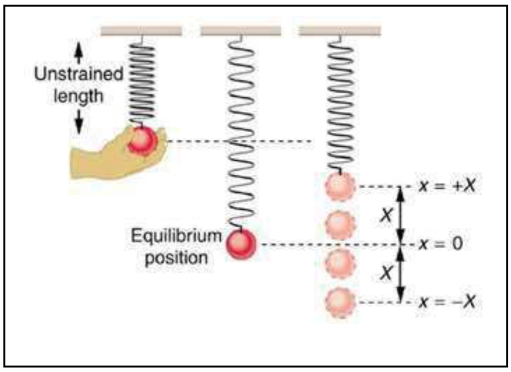
- Mass-Spring System
- Pendulum

- Simple harmonic motion (SHM) is a special kind of periodic motion occurs in mechanical system where net force acting on an object is proportional to the displacement of the object from its equilibrium position and the force is always directed towards the equilibrium position.
- In order for mechanical oscillation to occur, a system must posses two quantities: ***elasticity*** and ***inertia***.

- When the system is displaced from its equilibrium position, the *elasticity* provides a *restoring force* such that the system tries to return to equilibrium.
- The *inertia* property causes the system to *overshoot* equilibrium. This constant play between the elastic and inertia properties is what allows oscillatory motion to occur.

The natural frequency of the oscillation is related to the elastic and inertia properties

An oscillating system is a mass connected to a rigid foundation with a spring.



Example: Spring mass system

- An oscillating system is a mass connected to a rigid foundation with a spring.
- The spring constant k provides the elastic restoring force, and the inertia of the mass m provides the overshoot.

By applying Newton's second law $F=ma$ to the mass, one can obtain the equation of motion for the system:

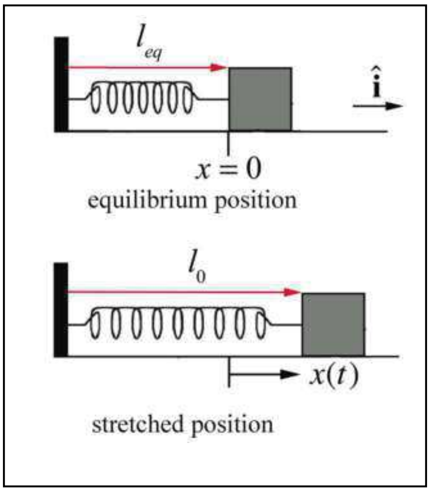
$$F = -kx \Rightarrow m\frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow \frac{d^2x}{dt^2} + \omega_o^2 x = 0$$

$$\omega_o = \sqrt{\frac{k}{m}}$$ is the natural frequency

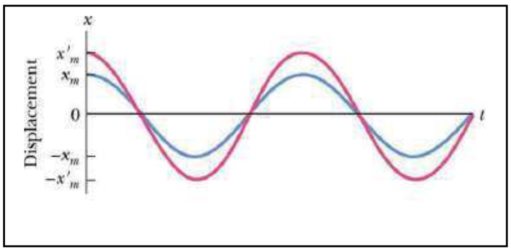
The solution of the wave equation

$$x(t) = x_m \cos(\omega_o t + \phi)$$

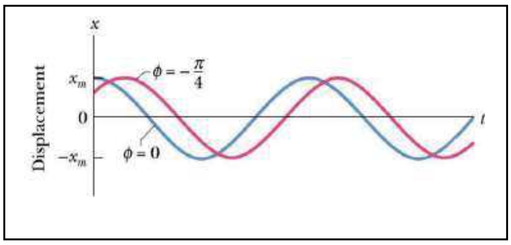
where x_m is the amplitude of the oscillation, and φ is the *phase constant* of the oscillation.



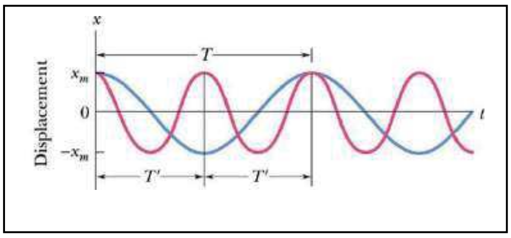
Waves with different amplitudes



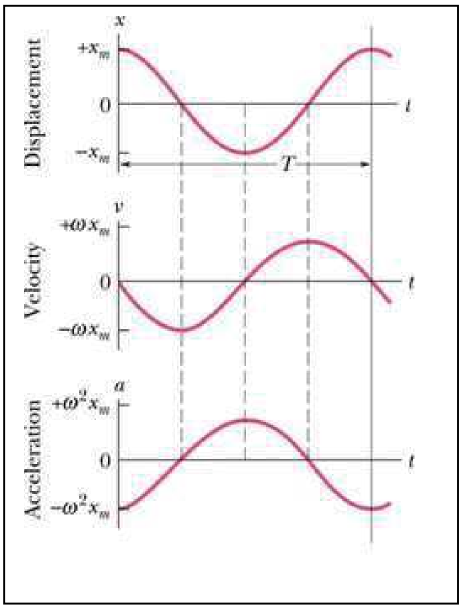
Waves with different phase



Waves with different time period



The period of the oscillatory motion is defined as the time required for the system to start one position, complete a cycle of motion and return to the starting position.



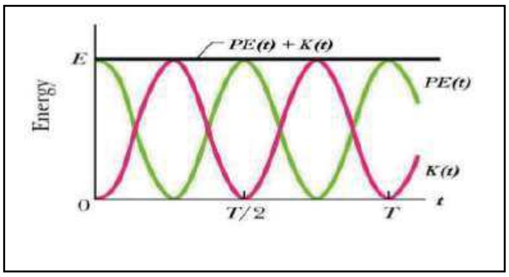
$$T = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{m}{k}}$$

$$v(t) = -\omega_o x_m \sin(\omega_o t + \phi)$$

$$a(t) = -\omega_o^2 x_m \cos(\omega_o t + \phi)$$

Energy in Simple Harmonic Motion

Kinetic energy (K) of the particle executing SHM



$$\mathrm{K} = \frac{1}{2}\mathrm{mv}^2$$

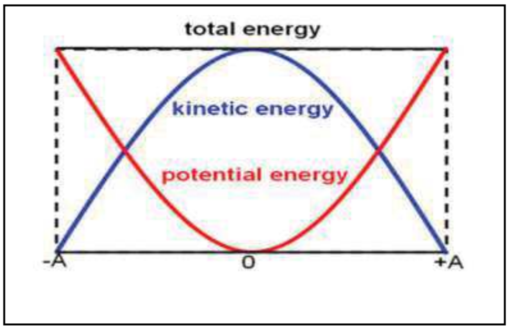
$$\mathrm{K} = \frac{1}{2}\mathrm{m}\omega^2\mathrm{A}^2\sin^2(\omega t + \varphi)$$

$$\mathrm{K} = \frac{1}{2}\mathrm{kA}^2\sin^2(\omega t + \varphi)$$

associated potential energy

$$U = \frac{1}{2} kx^2$$

$$U = \frac{1}{2} kA^2 \cos^2 (\omega t + \varphi)$$



Total energy, E, of the system is,

$$E = U + K = \frac{1}{2} kA^2$$

Throughout oscillation, KE continually being transformed into PE and *vice versa, but TOTAL ENERGY remains constant*

As the system oscillates, the total mechanical energy in the system trades back and forth between potential and kinetic energies.

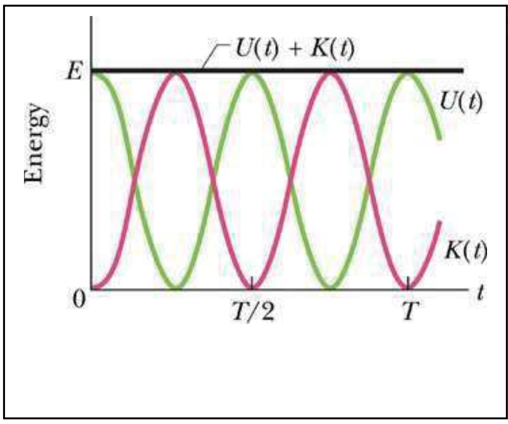
$$PE = \frac{1}{2} kx^2 = \frac{1}{2} kx_m^2 \cos^2(\omega_o t + \phi)$$

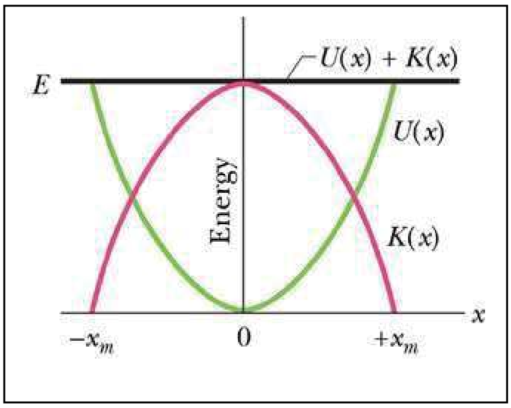
$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m\omega_o^2 x_m^2 \cos^2(\omega_o t + \phi)$$

$$PE + KE = \frac{1}{2} kx_m^2$$

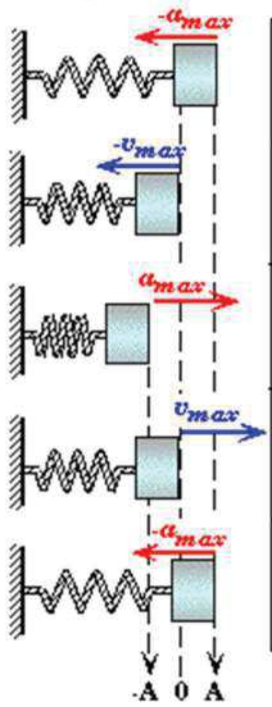
$$= \frac{1}{2} m\omega_o^2 x_m^2 = \frac{1}{2} mv_m^2$$

The total energy in the system, however, remains constant, and depends only on the spring constant and the maximum displacement (or mass and maximum velocity $v_m = \omega x_m$)



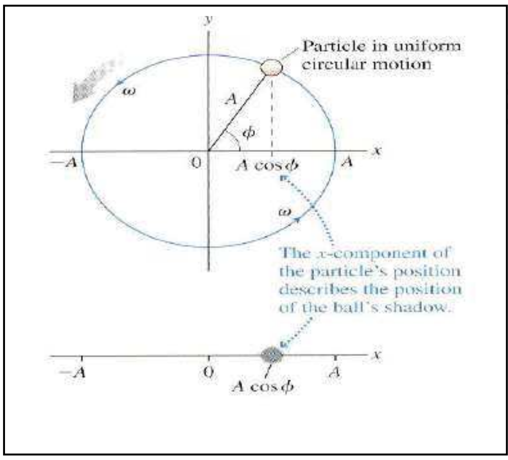


$\phi = \pi/2$



Time t	Position $x(t)$	Velocity $v(t)$	Accel. $a(t)$	KE	PE
0	x_{max}	0	$-a_{max}$	0	PE_{max}
$\frac{T}{4}$	0	$-v_{max}$	0	KE_{max}	0
$\frac{T}{2}$	$-x_{max}$	0	a_{max}	0	PE_{max}
$\frac{3T}{4}$	0	v_{max}	0	KE_{max}	0
T	x_{max}	0	$-a_{max}$	0	PE_{max}

Simple Harmonic Motion and Uniform Circular Motion



Displacement of oscillating object = projection on x-axis of object undergoing circular motion

$x(t) = A\cos\theta$

For rotational motion with angular frequency ω, displacement at time t:

$x(t) = A\cos(\omega t + \varphi)$

φ= angular displacement at t=0 (phase constant)

A = amplitude of oscillation (= radius of circle)

Velocity and Acceleration in Simple Harmonic Motion

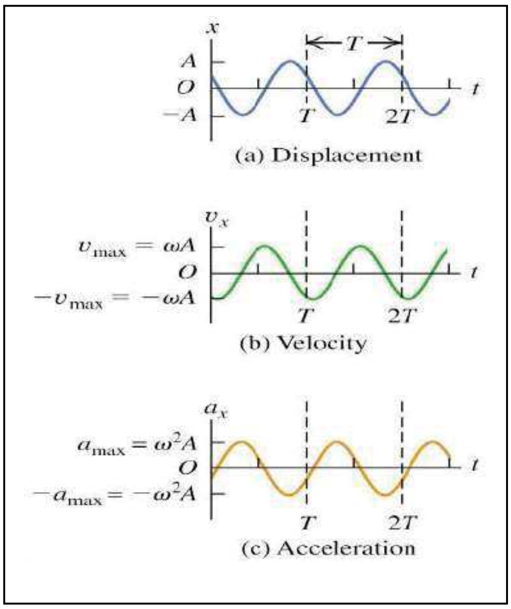
Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the latter motion takes place.

Displacement

$$x(t) = A\cos(\omega t + \varphi)$$

$$\text{Velocity}: v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \varphi)$$

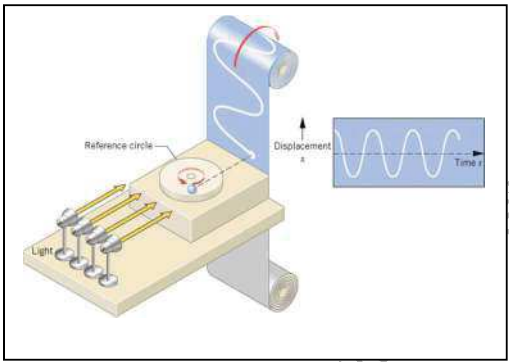
$$\text{Acceleration}: a(t) = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \varphi)$$



(a) Displacement
(b) Velocity
(c) Acceleration

Circular Motion

- Uniform Circular motion projected in one dimension is SHM



The ball mounted on the turntable moves in uniform circular motion, and its shadow, projected on a moving strip of film, executes simple harmonic motion.

Simple Pendulum

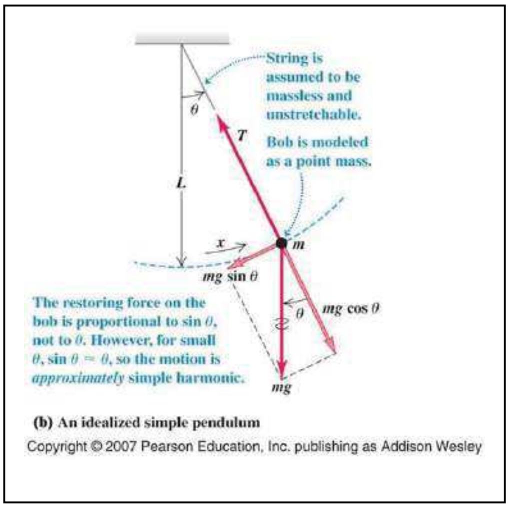
A pendulum consists of an object hanging from the end of a string or rigid rod pivoted about the point. The object is displaced (a small displacement; about 5-10^0) to one side and allowed to oscillate. If the object has negligible size and the string or rod is massless, then the pendulum is called a simple pendulum.

Restoring force

$$F = -mg\sin\theta$$

$$F \approx -mg\theta = -\text{mg}\frac{x}{L} \quad \text{for small angles}$$

$$\therefore \text{F} \propto x \quad \text{SHM motion}$$



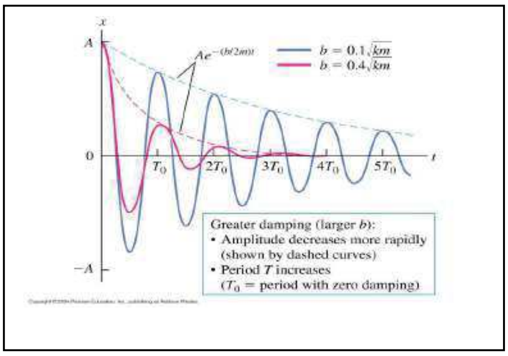
Damped Harmonic motion: Real oscillatory system

Have you ever thought why a simple pendulum or spring mass system comes to rest when they are kept in oscillatory motion. Ideally, the oscillatory motion should continue forever.

It is because of the resistance created by air, i.e. air works as damping medium which always apposes the motion or in other words we can say that the damping force is always in opposite direction to the restoring force.

In general, it is found that the damping force is proportional to the velocity of the oscillatory body.

Damped Oscillations

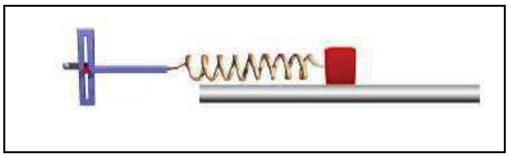


For damped oscillations, simplest case is when the damping force is proportional to the velocity of the oscillating object

Equation of motion:

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

Forced or Driven oscillation



- The natural frequency is the frequency at which it will oscillate if there is no driving and damping forces.

What is a wave?

A disturbance or variation that transfers energy progressively from point to point in a medium and that may take the form of an elastic deformation or of a variation of pressure, electric potential, temperature or more.

The Human Wave

The human wave is the disturbance (people jumping up and sitting back down), and it travels around the stadium. However, none of the individual people in the stadium are carried around with the wave as it travels - they all remain at their seats.

Waves in Everyday Life: Examples

- Disturbance produced in pond by throwing a stone creates ripples which move outward.
- Sound: Type of wave that moves through matter and then vibrates our eardrums so we can hear.
- Visible Light: Kind of wave that is made up of photons.
- Radio and TV Signals etc.

TYPES OF WAVES

(a) Mechanical waves
  - Requires medium for propagation
  - Governed by Newton's laws
  - Example: Water waves, sound waves, seismic waves, etc.

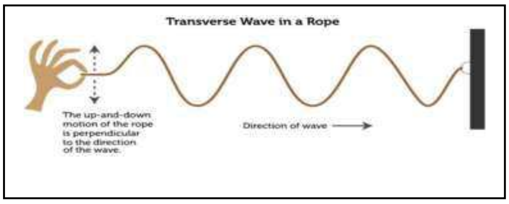
(b) Electromagnetic waves
  - Do not require any medium for their propagation
  - Example: Visible and ultraviolet light, Radio waves, Microwaves, X-rays etc.

(c) Matter waves
  - wave associated with the motion of a particle of atomic or subatomic size (electrons, protons, neutrons, other fundamental particles, and even atoms and molecules)

Waves differ from one another in the manner the particles of medium oscillate (or vibrate) with reference to the direction of propagation.

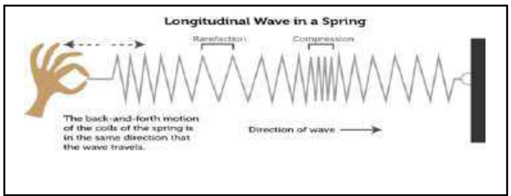
Transverse Wave

A wave in which the particles of the medium vibrate at right angles to the direction of propagation of wave, is called a transverse wave.



Longitudinal waves

A wave in which the particles of the medium vibrate in the same direction in which wave is propagating, is called a longitudinal wave.



Wave Parameters

- The amplitude A, is half the height difference between a peak and a trough.
- The wavelength λ, is the distance between successive peaks (or troughs).
- The period T, is the time between successive peaks (or troughs).
- The wave speed c, is the speed at which peaks (or troughs) move.
- The frequency ν, (Greek letter "nu") measures the number of peaks (or troughs) that pass per second.

A wave is a disturbance that travels from one location to another, and is described by a wave function that is a function of both space and time. If the wave function was sine function then the wave would be expressed by

$$y = A\sin(\omega t \pm kx)$$

The negative sign is used for a wave traveling in the positive x direction and the positive sign is used for a wave traveling in the negative x direction.

$$k = \frac{2\pi}{\lambda}$$
$$\omega = 2\pi \upsilon$$

Wave Speed

- The speed of a wave depends on the medium through which the wave moves.
- The speed, wavelength, and frequency are related.

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda v$$

- Speed of a Transverse Wave on Stretched String

$$v = \sqrt{\frac{T}{\mu}}$$

Where μ is linear mass density of a string, is the mass *m* of the string divided by its length *l*.

- Speed of a Longitudinal Wave Speed of Sound

$$v = \sqrt{\frac{B}{\rho}}$$

where B is bulk modulus and ρ is density of the medium.

- Speed of a longitudinal wave in an ideal gas

$$v = \sqrt{\frac{Y}{\rho}}$$

where *Y* is the Young's modulus of the material of the bar.

- Speed of a longitudinal wave in an ideal gas

$$v = \sqrt{\frac{P}{\rho}}$$

where *P* is the Pressure of the ideal gas. Above eqn. is known as Newton relation.