[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2425 **F-4** **Your Roll No................**

Unique Paper Code : 2352601

Name of the Course : **B.Tech. : Allied Course**

Name of the Paper : Numerical Methods

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.
3. All questions are compulsory.
4. Use of Scientific Calculator is allowed.

1. (a) Define the following terms

   (i) Order of method

   (ii) Floating point representation

   (iii) Local truncation error (6)

   (b) Perform five iterations by Bisection method to find the square root of 7. (6)

   (c) Apply Regula-Falsi method to $x^3 + x^2 - 3x - 3 = 0$ to determine an approximation to a root lying in the interval (1, 2). Perform four iterations. (6)

2. (a) Use Newton's method to solve the given non-linear system of equations:

   $f(x,y) = x^2 + y^2 - 1 = 0$

   $g(x,y) = x^2 - y = 0.$

P.T.O.

Take initial approximation $(x_0, y_0) = (0.5, 0.5)$ and perform two iterations. (6.5)

(b) Solve the following system of equations by using Gauss elimination (row pivoting) method.

$$3x - y + 2z = 7$$

$$x + y + 2z = 9$$

$$2x - 2y - z = -5$$ (6.5)

(c) Solve the following system of equation using Gauss Thomas method

$$2x - y = 1$$

$$-x + 2y - z = 0$$

$$-y + \frac{2}{3}z - t = -\frac{4}{3}$$

$$-z + 2t - u = 0$$

$$-t + 2u - v = 0$$

$$-u + 2v = 1$$ (6.5)

3. (a) Using Gauss-Jacobi method to solve given system of equations:

$$2x_1 - x_2 + x_3 = -1$$

$$x_1 + 2x_2 - x_3 = 6$$

$$x_1 - x_2 + 2x_3 = -3$$

Take initial approximation as $X^{(0)} = (0, 0, 0)^T$ and perform three iterations. (6)

(b) For the function $f(x) = \ln(x)$, construct Lagrange form of the interpolating polynomial for $f(x)$ passing through the points $(1, \ln 1)$, $(2, \ln 2)$ and $(3, \ln 3)$. Use the polynomial to estimate $\ln(1.5)$ and $\ln(2.4)$. (6)

(c) prove the following identities

(i) $\delta = \nabla(1 - \nabla)^{-1/2}$

(ii) $\nabla\left(\frac{1}{fi}\right) = \frac{-\Delta fi}{fifi+1}$ (6)

4. (a) Use Richardson extrapolation to estimate the first derivative of $y = \cos x.e^{-x}$ at x=1.0 using step size h=0.25. Employ the forward divided-difference formula to obtain the initial estimates. (6.5)

(b) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data

x	3	4.5	7	9
f(x)	2.5	1.0	2.5	0.5

Hence estimate the values of $f(3.5)$, $f'(3.5)$ and $f(8)$, $f'(8)$. What will be the estimate of higher order derivatives in piecewise linear interpolating polynomial (6.5)

(c) Derive backward divided difference formula of error O(h²). Using the derived formula estimate $f''(15)$ for the data given below with error O(h) and O(h²) respectively.

x	0	10	20	30
$f(x)$	56	94	108	120

(6.5)

5. (a) Derive Simpsons 1/3 rule formula. (6)

(b) Compute $\int_0^3 (x.e^{2x})dx$ using Gaussian Quadrature (6)

(c) Compute $\int_0^2 log(x^2 + 1)dx$ using

P.T.O.

(i) Trapezoidal Rule with n(number of sub-intervals)=4

(ii) Newton Cotes two point open formula (6)

6. (a) Solve the following initial value problem over the interval from x=0 to 2 where y(0)= 1

$$\frac{dy}{dx} = yx^3 - 1.5y$$

(i) Using Euler method with step size of 0.5.

(ii) Using Modified Euler method with step size of 0.5. (6.5)

(b) Describe the method(only) of fitting cubic spline to the following data

x	1	2	2.5	3
f (x)	1	5	7	8

(6.5)

(c) Apply finite-difference method to solve the problem:

$$\frac{d^2y}{d^2x} = (1 + x^2)y \qquad -1 \le x \le 1$$

with $y(-1) = y(1) = 1$, and $h = 0.2$. (6.5)

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