This question paper contains 3 printed pages. Your Roll No. ....

Sl. No. of Ques. Paper	: **6310**	**F–5**
Unique Paper Code	: **2341504**	
Name of Paper	: **Mathematical Physics – II**	
Name of Course	: **B. Tech. (Computer Science) (FYUP Scheme)**	
Semester	: **V**	
Duration	: **3 hours**	*Maximum Marks* : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do five questions in all. Question No. 1 is compulsory.

1. Do any ***five*** questions :

(a) Determine the order, degree and linearity of the differential equation:

$$\left(\frac{d^2 y}{d x^2}\right)^4 + \frac{d y}{d x} + 4 y = x$$

(b) What is Wronskian? Calculate the value of wronskian for:

$$x^n \quad \text{and} \quad x^n (\ln x)$$

(c) Prove the following property of Poisson Bracket:

$$[u v, w] = [u, w] v + u [v, w]$$

(d) Find the extreme points of the function:

$$f(x, y) = y^2 + 4 x y + 3 x^2 + x^3$$

(e) Solve:

$$\frac{d y}{d x} + \frac{n}{x} y = \frac{a}{x^n}$$

(f) Define generalised momenta for n-particle system, and find its time derivative.

(g) Form the differential equation whose only solutions are:

$$a_1, \; a_2 x e^{3x}, \; a_3 x^2 e^{3x}.$$

(h) Find the extremal of the integral :

$$\int_0^{\pi} \left(2 y \sin x - y'^2\right) d x, \text{ here } y' = \frac{d y}{d x}.$$ (5 × 3 = 15)

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2. Solve the following differential equations:

(a) $\frac{dy}{dx} = y \tan x - y^2 \sec x$ (6)

(b) $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$ (9)

3. Solve the following differential equations:

(a) $\frac{d^2y}{dx^2} - y = x \cos x$ (6)

(b) $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{(\ln x)^2}{x}$ (9)

4. (a) Solve the following differential equation

$$(x^4 + y^4)\,dx - x y^3\,dy = 0$$ (6)

(b) Using the method of variation of parameters, solve

$$(D^2 + 9)\,y = x \sin 3x \;;\; D \equiv \frac{d}{dx}$$ (9)

5. (a) Using the method of undetermined coefficients, solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x + x$$ (6)

(b) Solve the coupled differential equations:

$$\frac{dx}{dt} + 2x = \frac{dy}{dt} + 10 \cos t$$

$$\frac{dy}{dt} + 2y = 4e^{-2t} - \frac{dx}{dt}$$ (9)

6. (a) Find the equation of the shortest path between two points on the surface of sphere of radius a. (6)

(b) Using Lagrange's method of undetermined multiplier, find the maximum value of $u = x^p y^q z^r$ when the variables x, y, z are subjected to the condition $ax + by + cz = p + q + r$. (9)

7. (a) Find the Lagrangian corresponding to the Hamiltonian

$$H = \frac{p_x^{\,2}}{4a} + \frac{p_y^{\,2}}{4b} + k\,x\,y \qquad (6)$$

(b) Using Hamilton's equations of motion and the expression

$$L(q, \dot{q}) = p\,\dot{q} - H(q, p)$$

prove that : $p = \dfrac{\partial L}{\partial \dot{q}}$ and $\dot{p} = \dfrac{\partial L}{\partial q}$. (9)

8. (a) Show that

(i) $[q_j, H] = \dot{q}_j$

(ii) $[p_j, H] = \dot{p}_j,$ (6)

here, H denotes Hamiltonian and $1 \le j \le n$.

(b) Write the Lagrangian of the system of two masses 2m and m, shown below in Fig. (1). In this figure, y_1 and y_2 are the displacements of two masses from their equilibrium positions. Hence obtain the equations of motion of these two masses. (9)

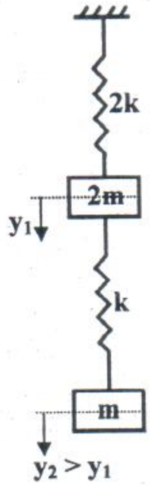

Fig. (1)

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