

Sl. No. of Ques. Paper : 1332

F-7

Unique Paper Code : 2341502

Name of Paper : Theory of Computation

Name of Course : B.Tech. Computer Science

Semester : V

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is of 35 marks and all its parts are compulsory.

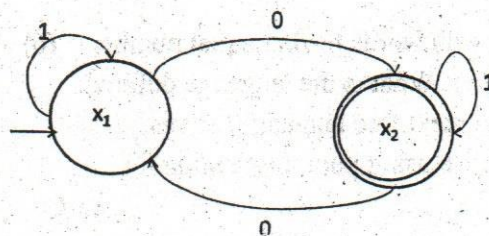
Attempt four questions from Q. Nos 2 to 7.

PART A

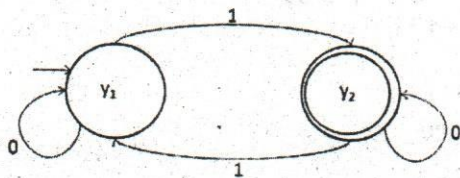
Note: For all the questions, consider the alphabet $\{a, b\}$ unless otherwise specified.

1. (a) Is $(S^*)^+ = (S^+)^*$ for all sets S ? Explain with an example. (2)
- (b) Consider the language PALINDROME and a string y over the given alphabet. Prove that if the string y^3 is in PALINDROME, then so is the string y . (3)
- (c) Give a regular expression for the language of all words that do not end in a double letter. (3)
- (d) Show that $(ab)^*a$ and $a(ba)^*$ define the same language. Give the set of strings representing the two languages. Give the first five strings generated in the lexicographic manner. (2)
- (e) Using pumping lemma for regular languages, show that the language $L = \{a^n b a^n \mid n \geq 0\}$ is not regular. (3)
- (f) Given two Finite Automata (FA): FA_1 and FA_2 find the machine for the intersection of the languages represented by these FA's. (4)

FA_1



P.T.O.



- (g) Create a Push Down Automata(PDA) for the language $L = \{ a^n S \}$, where S starts with b and length $(S) = n$. (4)

- (h) Find a Context Free Grammar(CFG) for the language defined by the regular expression a^*b^* . (2)

- (i) Show that the following CFG is ambiguous by finding a word with two distinct syntax trees: (3)

$S \rightarrow AA$

$A \rightarrow AAA \mid a \mid bA \mid Ab$

- (j) Convert the following CFG into CNF: (3)

$S \rightarrow aXX$

$X \rightarrow aS \mid bS \mid a$

- (k) Explain the working of the following Turing Machine(TM) (2)

$>R \xrightarrow{a \neq U} R \xrightarrow{b \neq U} R_U a R_U b$

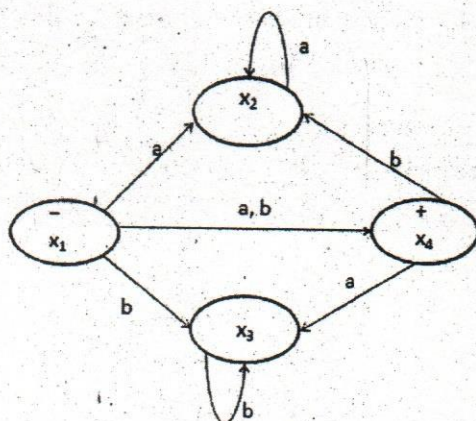
U represents the blank symbol.

- (l) Describe the Universal Turing Machine. (4)

PART B

- 2(a) Let language $L_1 = \text{EQUAL}$, the language with words having equal number of a 's and b 's and $L_2 = \{ a^n b^m a^n \mid m, n = 1, 2, \dots \}$. What is the language defined by the intersection of L_1 and L_2 ? Is it a context free language? If yes, construct a PDA for the language, else prove using pumping lemma for CFLs. (6)

- 2(b) Construct a CFG for the language $L = \{a^m b^n \mid m > n, m, n \geq 1\}$. (4)
- 3(a) Prove that regular languages are closed under complementation, i.e., if a language L is regular then L' (complement of L) is also regular. (3)
- 3 (b) For the following pair of regular languages find an FA that defines the difference, $L_1 - L_2$: (4)
- $L_1 = (a+b)^*c$
- $L_2 = b(a+b)^*c$
- $\Sigma = \{a, b, c\}$
- 3 (c) Build an FA that accepts the language of all strings of a's and b's such that the next to last letter is an a. (3)
- 4(a) Consider the homomorphism h from the alphabet $\{0, 1, 2\}$ to $\{a, b\}$. defined by: (4)
- $h(0) = ab, h(1) = b, h(2) = aa$
- i) What is $h(0210)$?
- ii) What is $h(2201)$?
- iii) If L is the language 1^*02^* , what is $h(L)$?
- 4(b) Give a PDA for the language with words of type $a^x b^y a^z b^w$, $x, y, z, w = 1, 2, 3$ (6)
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- $y > x$ and $z > w$ and $x + z = y + w$.
- 5(a) Convert the following NFA to DFA. (5)



- 5 (b) Write regular expression and construct a DFA for the following language of all words that have an even number of substrings ab in them. (5)

P.T.O.

- 6(a) Consider the following CFG in Chomsky Normal Form (CNF) (6)

 $S \rightarrow PQ$ $Q \rightarrow QS \mid b$ $P \rightarrow a$

Generate the derivation trees for the word i)abab ii)ababab

Consider S as the self embedded non terminal, trace the division of each word w into $uvxyz$ and $uvvxyyz$,

where $|u| + |z| \geq 0$, $|v| + |y| > 0$ and $|x| > 0$.

- 6(b) Which of the following could be configurations of a Turing Machine? (4)

Justify your answer.

i. $(q, \blacktriangleright aUaU, U, Ua)$

ii. (q, abc, b, abc)

iii. $(p, \blacktriangleright abc, a, e)$

iv. $(h, \blacktriangleright, e, e)$

(\blacktriangleright represents the left end symbol)

- 7 (a) Give a Turing Machine which computes the function $f(w) = ww$. (5)

- 7 (b) The language $H = \{ \langle M \rangle w : \text{Turing machine } M \text{ halts on input } w \}$ describes the halting problem. Prove that H is not recursive, i.e., the Halting problem is undecidable. (5)