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Sr. No. of Question Paper : 7801

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Your Roll No.....

Unique Paper Code : 2341201

Name of the Course : B.Tech. in Computer Science

Name of the Paper : Linear Algebra for Comp. Sc.

Semester : II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in all.
3. Question No. 1 is compulsory.
4. All the symbols have their usual meanings.

1. (a) Find the inverse of the following matrix using elementary row transformations

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

- (b) Describe the null space of the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 3 & 8 & 6 & 16 \end{bmatrix}$.

- (c) State and prove Cauchy-Schwarz's inequality.

- (d) Use Cramer's rule to solve $Ax = b$, where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

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- (e) Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (f) Let M be the space of all $n \times n$ matrices. Let $T: M \rightarrow M$ be a map such that

$$T(A) = \frac{A - A^t}{2}. \text{ Show that } T \text{ is linear. Also show that the kernel of } T \text{ consists of the space of symmetric matrices.}$$

- (g) The fraction of rental cars in a city starts at 0.02. The fraction outside that city is 0.98. Every month, 80% of the cars stay in the city (and 20% leave). Also 5% of the outside cars come in the city (95% stay outside). Estimate the fraction of cars coming and leaving the city at the end of 5 months. (5 \times 7 = 35)

2. (a) Find the complete solution to $Ax = b$, where

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 1 & 3 & 1 & 6 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix}$$

- (b) Write the LU-decomposition of the matrix

$$A = \begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 5 \end{bmatrix}$$

Also compute E_{21}^{-1} , E_{31}^{-1} , and E_{32}^{-1} to find L . (5,5)

3. (a) Define column space and null space of a matrix A . Let $Ax = b$ be the system of m linear equations in n unknowns. Prove that the column space and null space of A are the subspaces of R^m and R^n respectively.

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- (b) Find the bases and dimensions for all four fundamental subspaces of the following matrix A, namely, the column space of A, the null space of A, the row space of A and the null space of A^T . Also determine the rank of the matrix A.

$$A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \quad (5,5)$$

4. (a) Use Gram-Schmidt's orthogonalisation process to find the QR decomposition of the matrix

$$W = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 2 \end{bmatrix}$$

- (b) Find the pivots of the given matrix A and verify that $\det.A = \text{product of the pivots}$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} \quad (5,5)$$

5. (a) Diagonalise the following matrix A and hence find A^3 .

$$A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$$

- (b) Find the singular value decomposition (svd) of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$.

(5,5)

P.T.O.

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6. (a) Define a convex set. Draw the region in the xy -plane where $x + 2y \leq 6$, $2x + y \leq 6$ and $x \geq 0$, $y \geq 0$. Which corner minimizes the cost $c = 2x - y$?
- (b) Test whether the following matrix is positive definite or not.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad (5,5)$$

7. (a) Find the best line $b = C + Dt$ to fit $b = 0, 8, 8, 20$ at times $t = 0, 1, 3, 4$.
- (b) Let V be the vector space generated by the three functions $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = x^2$, where $x \in \mathbb{R}$. Let $D : V \rightarrow V$ be the derivative. What is the matrix of D w.r.t. the basis $\{f_1, f_2, f_3\}$? (5,5)

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