B.Tech. III. Sem. 2014

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1568**

Unique Paper Code : **2341303** **F-3**

Name of the Paper : **Discrete Structures**

Name of the Course : **B.Tech. in Computer Science**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A is compulsory.

Do any *four* questions from Section B.

Section A

1. (*a*) A collection of 10 electric bulbs contains 3 defective ones :

   (*i*) In how many ways can a sample of four bulbs be selected ?

   (*ii*) In how many ways can a sample of four bulbs be selected which contains two good bulbs and 2 defective ones ? 1+2

   (*b*) Suppose $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$ then show that $(f_1(x) + f_2(x))$ is $O(\max(g_1(x), g_2(x)))$. 3

   (*c*) Evaluate the sum : 3

$$\frac{\sum_{k=1}^{\infty} k^2}{2^k}.$$

P.T.O.

(*d*) Show that : 2

$$\bar{p} \rightarrow (q \rightarrow r) \text{ and } q \rightarrow (p \vee r)$$

are logically equivalent.

(*e*) Determine the discrete numeric function of the generating function : 4

$$A(Z) = Z^2/(4 - 4Z + Z^2).$$

(*f*) Prove that every bipartite graph is 2-colorable. 2

(*g*) Using master theorem, find the solution to the recurrence : 3

$$4T(n/2) + n^2 = T(n).$$

(*h*) Consider a set A = {2, 7, 14, 28, 56, 84} and the relation $a \leq b$ if and only if a divides b. Give the Hasse diagram for the Partial order set (A, ≤). 3

(*i*) How many edges are there in an undirected graph with two vertices of degree 7, four vertices of degree 5, and four vertices of degree 6 ? 2

(*j*) Show that a full m-ary balanced tree of height h has more than m^{h-1} leaves. 3

(*k*) Let $|A| = n$ and $|B| = m$ where $m > n$. Give the number of one-to-one functions, $f : A \rightarrow B$ that can be defined ? 2

(*l*) Show that for a graph to be planar it should at least one vertex of degree 5 or less. 3

(*m*) Write the contrapositive and inverse of the following statements : 2

If you try hard, then you will win.

Section B

2. (*a*) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. 4

(*b*) Let X = {1, 2, 3, 4, 5, 6}, and define a relation R on X as R = {(1, 2), (2, 1), (2, 3), (3, 4), (4, 5), (5, 6)}. Find the reflexive and transitive closure of R. 3

(*c*) Prove that $n^3 - n$ is divisible by 3 for any integer $n \geq 0$. 3

3. (*a*) Let '*a*' be a numeric function such that : 4

$$a_r = \begin{pmatrix} 2 & 0 \leq r \leq 3 \\ 2^{-r} + 5 & r \geq 4 \end{pmatrix}$$

(*i*) Determine Δa

(*ii*) ∇a.

(*b*) Find the total solution of the recurrence relation : 4

$$a_n + 4a_{n-1} = 7, \text{ where } a_0 = 3.$$

(*c*) How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points ? 2

P.T.O.

4. (a) Draw the graph K_6 and answer the following questions : 1+3

(i) What is the degree of each vertex ?

(ii) Is K_6 planar ?

(b) A connected planar graph G has 10 vertices each of degree 3. Into how many regions does a representation of this planar graph split the plane ? 4

(c) How many leaves does a regular (full) 3-ary tree with 100 vertices have ? 2

5. (a) Draw graphs each having six vertices such that : 4

(i) Graph is Hamiltonian but not Eulerian

(ii) Graph is Eulerian but not Hamiltonian.

(b) Show that the sum of degree of all vertices in G is twice the number of edges in G. 3

(c) What is the condition for $K_{m,n}$ to have an Euler path or circuit ? Justify your answer. 3

6. (a) Use the insertion sort algorithm to sort the list 2, 14, 9, 13, 12. 3

(b) Determine whether each of the functions 2^{n+1} and 2^{2n} is $O(2^n)$. 4

(c) Using substitution method, prove that $T(n)$ is $O(n \lg n)$ given that : 3

$$T(n) = 2T\left(\frac{n}{2}\right) + n.$$

7. (a) Show that $(p \to q) \wedge (r \to q) \Leftrightarrow (p \vee r) \to q$ are logically equivalent using the laws of logical equivalences. 4

(b) Show that :

$$(p \wedge q) \to (p \vee q)$$

is a tautology with the help of truth table. 2

(c) Show that the premises "It is not sunny this afternoon and it is colder than yesterday;" "We will go swimming only if it sunny;" "If we do not go swimming, then we will take a canoe trip"; and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion—"We will be home by sunset". 4