Logarithms

Why logarithms:

We know that $32 = 2^5$. ie., 32 can be represented as a power of 2. But how do we represent 32 as some power of 10?

This is where logarithms comes. 32 when represented a power of 10 is equal to $\log_{10}32 = 1.5051$. i.e., $32 = 10^{1.5051}$

Properties of Logarithms:

1. $\log_a 1 = 0$ because 0 is the power to which a must be raised to obtain 1.
2. $\log_a a = 1$ because 1 is the power to which a must be raised to obtain a.
3. $\log_a a^N = N$ because N is the power to which a must be raised to obtain a^N
4. $a^{\log_a b} = b$ because $\log_a b$ is the power to which a must be raised to obtain b.
5. $a^{\log_b c} = c^{\log_b a}$
6. $\log_{a^b} M = \frac{1}{b}\log_a M$
7. $\log_b M = \frac{\log_a M}{\log_a b}$
8. $\log_a b = \frac{1}{\log_b a}$
9. $\log_a b.\log_b c = \log_a c$

Laws of logarithm:

1. Product rule: $\log_a MN = \log_a M + \log_a N$
2. Division rule = $\log_a \frac{M}{N} = \log_a M - \log_a N$

Remember:

$\log_a(b + c) \neq \log_a b + \log_a c$

Note: When no base is given we generally assume the base as 10. These are called common logarithms

When "e" is a base then we call them as natural logarithms.

Characteristic of logarithm:

The characteristic of the logarithm of a number greater than one is one less than the number of digits in it.

Eg: characteristic of 98765 = 4 so total digits in the given number is 5.

We know that log 100 = $\log 10^2 = 2\log 10 = 2.00$. As the characteristic of 100 is 2, then total digits in 100 are 3.

log 10 = $\log 10^1 = 1\log 10 = 1$ So total digits are 2 in 10.

log 99 = 1.9956 so total digits are 1 + 1 = 2.

Working Rule to find the number digits in a^b format number:

1. Calculate the logarithm (you will get some positive number)

2. Adding one to the intezer part will give you the number of digits in that original number

Solved Example 1: (*Important model*)

How many digits are contained in the number 2^{100}

Sol: $\log 2^{100}$ = 100 x log 2 = 100 x 0.3010 = 30.10

Number of digits in 2^{100} are 30 + 1 = 31

To determine the characteristic of the logarithm of a decimal fraction: (Numbers between 0 to 1)

Look at this example:

Find the total zeroes after he decimal point of the expression 2^6

We know that 2^6 = $\frac{1}{64}$ = 0.015625

log $\frac{1}{64}$ = -1.806

Now when you calculate Antilog of -1.806 using calculator, you will get 0.0156.

But if you want to use antilog tables, you have to follow this procedure.

Now log $\frac{1}{64}$ = log $\frac{1}{2^6}$ = log 2^{-6} = -6 log 2.

We know that log 2 = 0.301

Now log $\frac{1}{64}$ = -6 × 0.301 = -1.806.

Important: Now if you look at the antilog table for 0.80 and 6, you will get wrong answer. Why? Because -1.806 = -1 + (-0.806)

But mantissa is always positive.

-1.806 should be written as -2 + (1 - 0.806) = -2 + 0.194

Now when you look at the anti log table 0.194 gives you 1563.

So characteristic is 2.

That is the characteristic of the logarithm of a decimal fraction is one more than than the number of zeroes immediately after the decimal point and is negative.

Working Rule to find the number of zeroes in a decimal number:

1. Calculate the logarithm (you will get some negative number)

2. Subtract the decimal part from one and increase the integer part by 1 to make mantissa positive

3. Number of zeroes of that number = Integer part - 1

Solved Example 2: (*Important model*)

How many zeroes are there between the decimal point and the first significant digit in $(\frac{1}{2})^{1000}$

$\log\left(\frac{1}{2}\right)^{1000}$ = 1000 × log (1/2) = 1000 × -0.30102 = -301.02

But in logarithms the decimal point should be positive. (By using

- 301.02 = - 301 + -0.02 = -302 + (1 - 0.02) = $\overline{302}$.98

So number of zeroes are 302 - 1 = 301

Solved Example 3:

$\frac{1}{1+\log_a bc}+\frac{1}{1+\log_b ca}+\frac{1}{1+\log_c ab}=$

a. 0 b. 3

c. 2 d. 1

Answer: d

Explanation:

$\frac{1}{1+\log_a bc}+\frac{1}{1+\log_b ca}+\frac{1}{1+\log_c ab}=$

$\frac{1}{\log_a a+\log_a bc}+\frac{1}{\log_b b+\log_b ca}+\frac{1}{\log_c c+\log_c ab}$

$=\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}=$

$=\log_{abc}a+\log_{abc}b+\log_{abc}c=\log_{abc}abc=1$

Solved Example 4:

The value of $(yz)^{\log y-\log z}\times(zx)^{\log y-\log x}\times(xy)^{\log x-\log y}$

a. 2 b. 1

c. 0 d. 3

Answer: b

Explanation:

Assume K = $(yz)^{\log y-\log z}\times(zx)^{\log y-\log x}\times(xy)^{\log x-\log y}$

Taking log on both sides

Log K = log $((yz)^{\log y-\log z}\times(zx)^{\log y-\log x}\times(xy)^{\log x-\log y})$

$=\log(yz)^{\log y-\log z}+\log(zx)^{\log y-\log x}+\log(xy)^{\log x-\log y}$

$=(\log y-\log z)\log(yz)+(\log z-\log x)\log(zx)+(\log x-\log y)\log(xy)$

$=(\log y-\log z)(\log y+\log z)+(\log z-\log x)(\log z+\log x)+(\log x-\log y)(\log x+\log y)=0$

log K = 0⇒ K = 1

Solved Example 5:

$\left(\frac{x+y}{3}\right)=\frac{1}{2}$ (logx + logy) then $\left(\frac{x}{y}+\frac{y}{x}\right)$ is

a. 5 b. 7

c. 9 d. 0

Answer: b

Explanation:

As the L.H.S does not have any log, first we try to remove log from the given equation.

$\log\left(\frac{x+y}{3}\right)=\frac{1}{2}$(logx + logy)

$\Rightarrow 2\log(\frac{x+y}{3})$ = log xy

$\Rightarrow \log(\frac{x+y}{3})^2 = \log xy$

$\Rightarrow x^2 + y^2 + 2xy = 9xy$

$\Rightarrow x^2 + y^2 = 7xy$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = 7$

Solved Example 6:

The value of $7\log_a \frac{16}{15} + 5\log_a \frac{25}{24} + 3\log_a \frac{81}{80}$ is

a. $\log_a 5$

b. $\log_a 3$

c. $\log_a 2$

d. $\log_a 0$

Answer: c

Explanation:

$7\log_a \frac{16}{15} + 5\log_a \frac{25}{24} + 3\log_a \frac{81}{80} = 7\log_a [\frac{3^4}{3 \times 5}] + 5\log_a [\frac{5^2}{3 \times 2^3}] + 3\log_a [\frac{3^4}{5 \times 2^4}]$

$= 7[4\log_a 2 - \log_a 3 - \log_a 5] + 5[2\log_a 5 - \log_a 3 - 3\log_a 2] + 3[4\log_a 3 - \log_a 5 - 4\log_a 2]$

$= \log_a 2$

Solved Example 7:

If $x = \log_{2a} a, y = \log_{3a} 2a, z = \log_{4a} 3a$ then $(x-2)yz$ =

a. -1

b. -2

c. 1

d. 2

Answer: a

Explanation:

(x-2)yz = xyz - 2yz

Substituting values from the given values from above,

$\log_{2a} a . \log_{3a} 2a . \log_{4a} 3a - 2\log_{3a} 2a . \log_{4a} 3a = \log_{4a} a - 2\log_{4a} 2a$

$\Rightarrow \log_{4a} a - \log_{4a} (2a)^2 = \log_{4a} [a/(2a)^2]$ (Division Rule)

$\Rightarrow \log_{4a} [1/4a] = \log_{4a} 4a^{-1} = -1$

Solved Example 8:

If $a = \log_4 5$ and $b = \log_5 6$ then $\log_2 3$ = ?

a. 1-2ab

b. 1+2ab

c. 2ab - 1

d. (a-b) / (a+b)

Answer: c

Explanation:

To solve questions like these, first observe the question asked. The given question contains 2 and 3. If we remove 5 from the given values of a and b, we are left with 4 and 6.

$ab = \log_4 5.\log_5 6 = \log_4 6 = \frac{1}{2}\log_2 6$

By product rule, $\log_2 6 = \log_2(3 \times 2) = \log_2 3 + \log_2 2$

$\frac{1}{2}(\log_2 3 + \log_2 2) = \frac{1}{2}(\log_2 3 + 1)$

$\Rightarrow ab = \frac{1}{2}(\log_2 3 + 1)$

$\Rightarrow 2ab = \log_2 3 + 1$

$\Rightarrow \log_2 3 = 2ab - 1$

Solved Example 9:

If $a = \log_{105} 7, b = \log_7 5$ then $\log_{35} 105$ =

a. ab

b. (b+1)a

c. 1/ab

d. 1/a(b+1)

Answer: d

Explanation:

$ab = \log_{105} 7.\log_7 5 = \log_{105} 5$

Now $\log_{35} 105 = \frac{1}{\log_{105} 35} = \frac{1}{\log_{105} 5 + \log_{105} 7} = \frac{1}{ab + a} = \frac{1}{a(b+1)}$