Sequences

Sequences are important question types. By practicing a few questions one can easily crack these questions with ease. Study the following.

1. Find the sum to n terms of series 1.2.3 + 2.3.4 + 3.4.5+ ....

a. (n+1)(n+2)(n+3)/3 b. n(n+1)(2n+2)(n+2)/4

c. n(n+1)(n+2) d. n(n+1)(n+2)(n+3)/4

Sol:

The general term of the above series = n(n+1)(n+2)

Sum = $\sum n(n+1)(n+2) = \sum n^3 + 3n^2 + 2n = \sum n^3 + 3\sum n^2 + 2\sum n$

(Recap the formulas for the sum of first n natural numbers, and its squares and its cubes)

$\Rightarrow \dfrac{n^2(n+1)^2}{4} + 3.\dfrac{n(n+1)(2n+1)}{6} + 2.\dfrac{n(n+1)}{2}$

$\Rightarrow n(n+1)[\dfrac{n^2+n+4n+2+4}{4}]$

$\Rightarrow n(n+1)[\dfrac{n(n+1)}{4} + \dfrac{2n+1}{2} + 1]$

$\Rightarrow n(n+1)[\dfrac{n^2+5n+6}{4}]$

$\Rightarrow n(n+1)[\dfrac{(n+2)(n+3)}{4}]$

So option 4.

Alternate method:

Above mentioned method to be used only when the sum upto a certain terms was asked. Say upto 20 terms. If infinite terms sum was asked, Simply we check the answer options. Take the sum upto 3 terms.

6 + 24 + 60 = 90.

Now substitute n = 3 in the answer options. Only Option 4 satisfies.

2. $\sqrt{1+\dfrac{1}{1^2}+\dfrac{1}{2^2}} + \sqrt{1+\dfrac{1}{2^2}+\dfrac{1}{3^2}} + \dots\dots\dots \sqrt{1+\dfrac{1}{2007^2}+\dfrac{1}{2008^2}} =$

a. $2008 - \dfrac{1}{2008}$ b. $2007 - \dfrac{1}{2007}$

c. $2007 - \dfrac{1}{2008}$ d. $2008 - \dfrac{1}{2007}$

Sol:

The clue to solve the problem lies in the options provided. Observe that there are total 2007 terms given. and options are in 2007's and 2008's

Consider only first term:

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} = \sqrt{1+1+\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} = 2-\frac{1}{2}$$

Consider first two terms:

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}$$

$$\Rightarrow \frac{3}{2}+\sqrt{1+\frac{1}{4}+\frac{1}{9}} = \frac{3}{2}+\sqrt{\frac{49}{36}} = \frac{3}{2}+\frac{7}{6} = \frac{8}{3} = 3-\frac{1}{3}$$

Similarly: $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \ldots\ldots\ldots \sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$ = 2008 - $\frac{1}{2008}$

3. S= $\frac{1}{1+1^2}+\frac{1}{2+2^2}+\frac{1}{3+3^2}+\ldots\ldots$ What is the value of S?

(1) 2 (2) 1.5 (3) 1 (4) The sum is not finite.

Sol: Give S= $\frac{1}{1+1^2}+\frac{1}{2+2^2}+\frac{1}{3+3^2}+\ldots\ldots$

$$\Rightarrow \frac{1}{1(1+1)}+\frac{1}{2(1+2)}+\frac{1}{3(1+3)}+\ldots\ldots$$

$$\Rightarrow \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\ldots$$

$$\Rightarrow (\frac{1}{1}-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}-\frac{1}{4})+\ldots\ldots$$

$$\Rightarrow \frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots$$

4. N, the set of natural numbers, is partitioned into subsets S_1 = {1}, S_2 = {2, 3}, S_3 ={4, 5, 6}, S_4 = {7, 8, 9, 10} ...... Find the sum of the elements in the subset S_{50}

Sol: Observe that the last term of every subset is 1, 1 +2, 1+2+3, 1+2+3+4, ...

So last term is written as $\frac{n(n+1)}{2}$

Now nth subset has n elements in Arithmetic Progression with common difference 1.

To get the first term we have to substract (n-1) from the last term. (take S4, if we substract 3 from 7, we get the first term of that subset)

So first term = $\frac{n(n+1)}{2}-(n-1) = \frac{n^2+n-2n+2}{2} = \frac{n^2-n+2}{2}$

So sum = $\frac{n}{2}[\frac{n^2-n+2}{2}+\frac{n(n+1)}{2}] = \frac{n(n^2+1)}{2}$

So S_{50} = $\frac{50(50^2+1)}{2} = 25\times 2501$ = 62625

5. Find the value of $\frac{2}{3}+\frac{16}{3^2}+\frac{78}{3^3}+\frac{320}{3^4}+\frac{1210}{3^5}+\ldots\ldots\ldots$

Sol:

$$\frac{2}{3}+\frac{16}{3^2}+\frac{78}{3^3}+\frac{320}{3^4}+\frac{1210}{3^5}+\ldots\ldots\ldots = 1(1-\frac{1}{3})+2(1-\frac{1}{3^2})+3(1-\frac{1}{3^3})+\ldots\ldots\ldots$$

(1 + 2 + 3 +........10) - ($\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \ldots\ldots$

[$\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \ldots\ldots$ is a Arithmetico geometric progression

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$ for AGP]

Here a = 1/3; d = 1; r = 1/3

$$\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \ldots\ldots = \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{1.\frac{1}{3}}{(1-\frac{1}{3})^2} = \frac{5}{4}$$

$\Rightarrow$ 55 - $\frac{5}{4} = \frac{215}{4}$