

Sets

Problems on sets some times may not be straight forward. We need to apply very strong logic to dig out the underlying the mystery. Study the following problems. First problem is easy.

New age consultants have three consultants Gyani, Medha, and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha involved. All three consultants are involved together in 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with with Medha but without Gyani and 3 projects with Gyani but without Medha. The total number of projects for New age consultants is one less than twice the number of projects in which more than one consultant is involved.

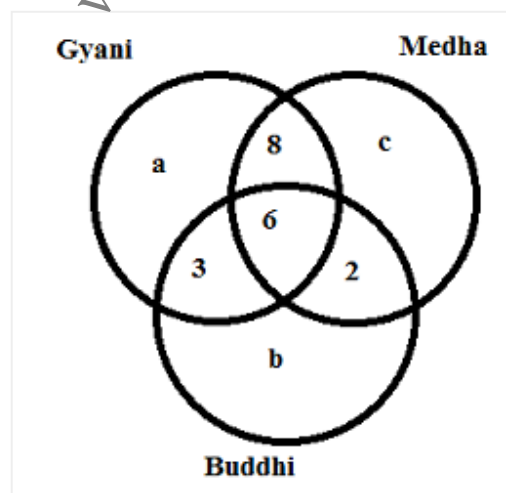
1. What is the number of projects in which Gyani alone is involved?
2. What is the number of projects in which Medha alone is involved?

Step 1: It was given that all the three consultants are involved in 6 projects together. So put 6 where all the three circles intersect

Step 2: Buddhi has 2 projects with Medha but without Gyani i.e., Medha and Buddhi alone worked in 2 projects.

Step 3: Also given, Buddi worked on 3 projects with Gyani but without Medha. i.e., Buddi adn Gyani alone worked on 3 projects.

Step 4: Gyani worked with Medha in 14 projects. These two worked together along with Buddhi in 6 projects. So Gyani and Medha alone worked on $14 - 6 = 8$ Projects.



Lets us assume, Gyani, Buddhi and Medha alone worked on a, b, c projects.

It was given that The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha involved.

$$\Rightarrow a + b = 8 + 6 + 2 + c$$

$$\Rightarrow a + b = 16 + c \text{ ----- (1)}$$

Also given The total number of projects for New age consultants is one less than twice the number of projects in which more than one consultant is involved.

The projects in which more than 1 consultant involved = $6 + 3 + 2 + 8 = 19$

Total projects = $a + b + c + 8 + 3 + 2 + 6 = a + b + c + 19$

$$\Rightarrow 2 \times 19 - 1 = a + b + c + 19$$

$$\Rightarrow a + b + c = 18$$

$$\Rightarrow a + b = 18 - c$$

Substituting this value in equation (1) we get

$$18 - c = 16 + c \Rightarrow c = 1$$

So from equation (1) we get $a + b = 17$.

Question 1: Gyani alone involved in "a" projects. But we cannot determine uniquely the value of "a"

Question 2: Medha alone involved in "c" projects. We calculated $c = 1$

Help Distress (HD) is an NGO involved in providing assistance to people suffering from natural disasters. Currently, it has 37 volunteers. They are involved in three projects: Tsunami Relief (TR) in Tamil Nadu, Flood Relief (FR) in Maharashtra, and Earthquake Relief (ER) in Gujarat. Each volunteer working with Help Distress has to be involved in at least one relief work project.

*A Maximum number of volunteers are involved in the FR project. Among them, the number of volunteers involved in FR project alone is equal to the volunteers having additional involvement in the ER project.

*The number of volunteers involved in the ER project alone is double the number of volunteers involved in all the three projects.

*17 volunteers are involved in the TR project.

*The number of volunteers involved in the TR project alone is one less than the number of volunteers involved in ER Project alone.

*Ten volunteers involved in the TR project are also involved in at least one more project.

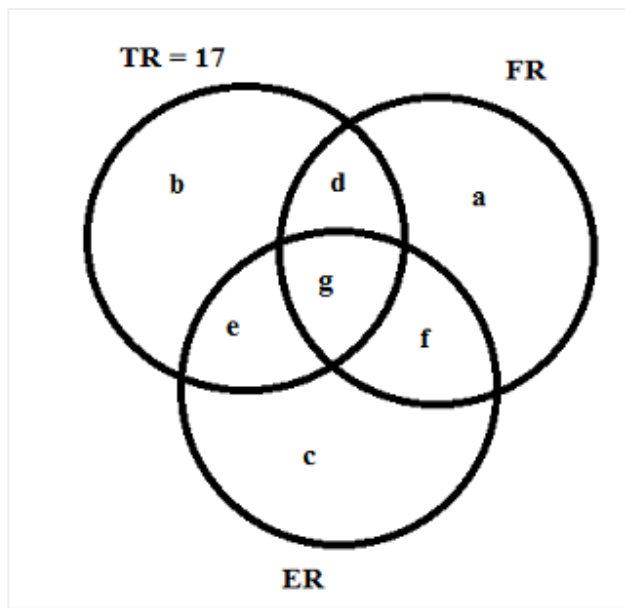
Find the minimum number of volunteers involved in both FR and TR projects, but not in the ER project is:

(1) 1

(2) 3

(3) 4

(4) 5



Statement 1: the number of volunteers involved in FR project alone is equal to the volunteers having additional involvement in the ER project.

$$a = g + f$$

Statement 2: The number of volunteers involved in the ER project alone is double the number of volunteers involved in all the three projects.

$$c = 2g$$

Statement 3: 17 volunteers are involved in the TR project.

$$b + d + g + e = 17$$

Statement 4: The number of volunteers involved in the TR project alone is one less than the number of volunteers involved in ER Project alone.

$$c - b = 1$$

Statement 5: Ten volunteers involved in the TR project are also involved in at least one more project.

$$d + g + e = 10$$

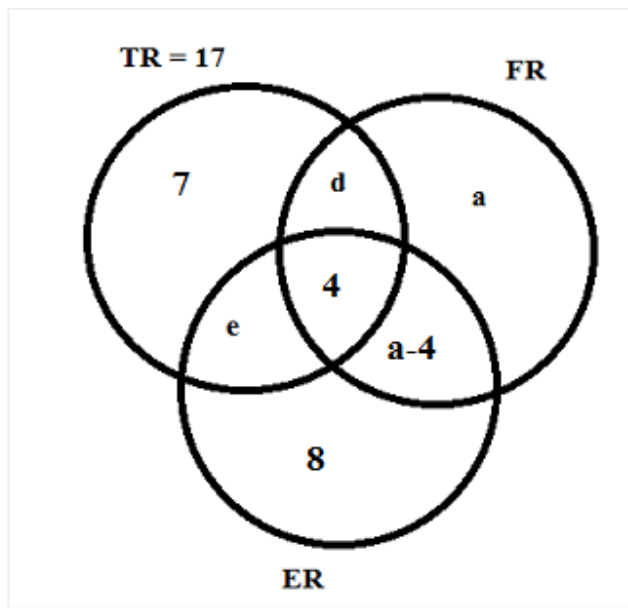
From statements 3 and 5 we get $b = 7$

From statement 4, we get $c = 8$

From statement 2, we get $g = 4$

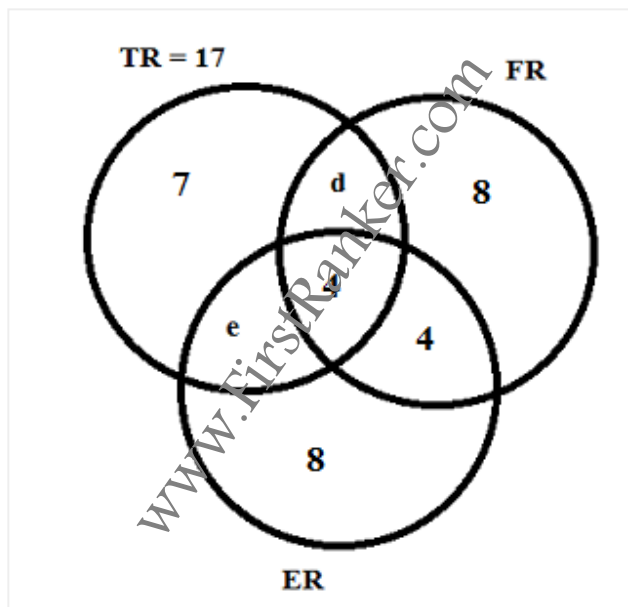
From statement 1 we get $f = a - 4$

Substituting the above values we get the diagram as



But the total volunteers are equal to 37

$$\text{So } 17 + a + a - 4 + 8 = 37 \Rightarrow a = 8$$



From the above diagram we know that $d + e = 6$, Also given that maximum number of volunteers are involved in FR.

$$\text{i.e., } FR > TR \Rightarrow FR > 17$$

$$FR > ER \Rightarrow 16 + d > 16 + e \Rightarrow d > e$$

Possible value of d and e in this case are (4, 2), (5, 1), (6, 0)

From the above analysis d takes minimum value 4.

Additional Questions based on the above set:

Which of the following additional information would enable to find the exact number of volunteers involved in various

projects?

- (1) Twenty volunteers are involved in FR.
- (2) Four volunteers are involved in all the three projects.
- (3) Twenty three volunteers are involved in exactly one project.
- (4) No need for any additional information.

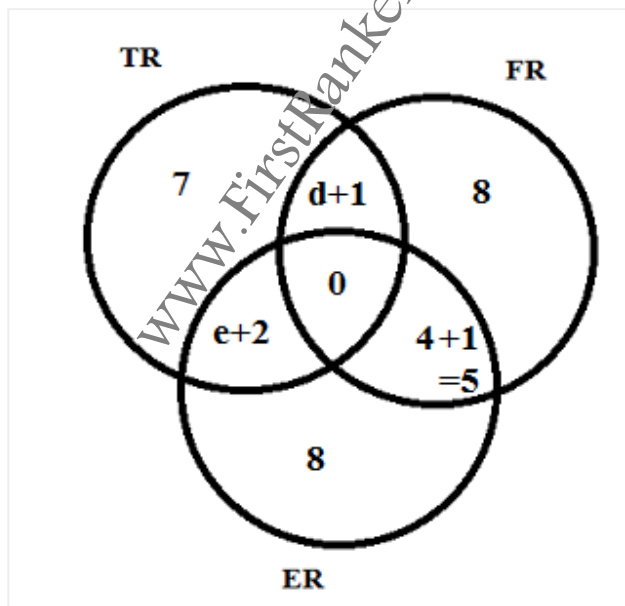
Ans: If we know the value of d , we can easily figure out the value of e . so statement 1 is required

After some time, the volunteers who were involved in all the three projects were asked to withdraw from one project. As a result, one of the volunteers opted out of the TR project, and one opted out of the ER project, while the remaining ones involved in all the three projects opted out of the FR project.

Which of the following statements, then, necessarily follows?

- (1) The lowest number of volunteers is now in TR project.
- (2) More volunteers are now in FR project as compared to ER project.
- (3) More volunteers are now in TR project as compared to ER project.
- (4) None of the above

Due to this new arrangement the number of workers working on each project changes



Number of workers in TR project = 16

Number of workers in FR project = 14 + d

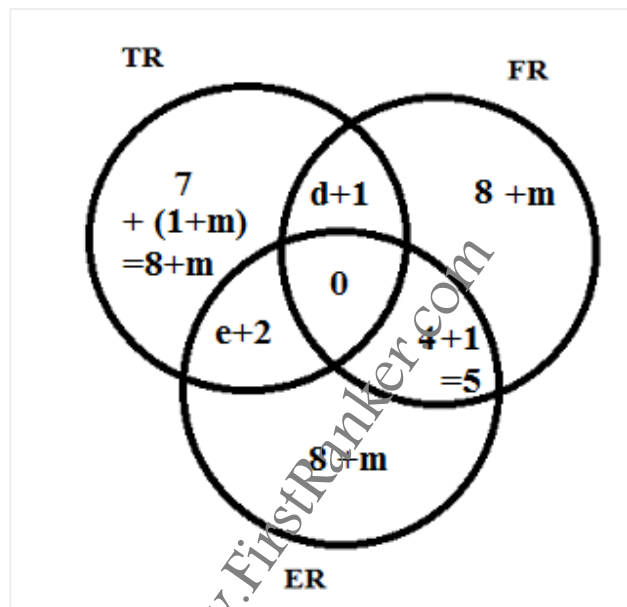
Number of workers in ER project = 15 + e

Considering the possible value of d and e , d takes minimum 4 so $FR > ER$

(But it is not clear in this questions the condition that FR has greater number of volunteers holds good or not. If that is not the case, answer for this question cannot be determined.)

After the withdrawal of volunteers, some new volunteers joined the NGO. Each one of them was allotted only one project in a manner such that, the number of volunteers working in one project alone for each of the three projects became identical. At that point, it was also found that the number of volunteers involved in FR and ER projects was the same as the number of volunteers involved in TR and ER projects. Which of the projects now has the highest number of volunteers?

- (1) ER
- (2) FR
- (3) TR
- (4) Cannot be determined



Given that volunteers working with only one project is same for all the three projects. So add $m + 1$ to TR and m to FR and ER.

Also given that volunteers working in ER and TR are equal to ER and FR. So $e + 2 = 5 \Rightarrow e = 3$.

As we know $d + e = 6 \Rightarrow d = 3$

Now number of volunteers in each of the projects given below

$$TR = 8 + m + (3 + 1) + (3 + 2) = 17 + m$$

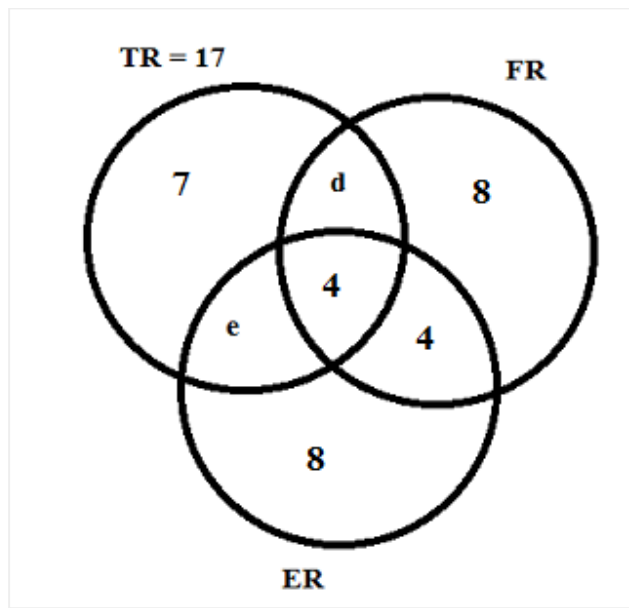
$$FR = 8 + m + 5 + (3 + 1) = 17 + m$$

$$ER = 8 + m + (3 + 2) + 5 = 18 + m$$

So ER has the highest number of volunteers.

Additional Observations:

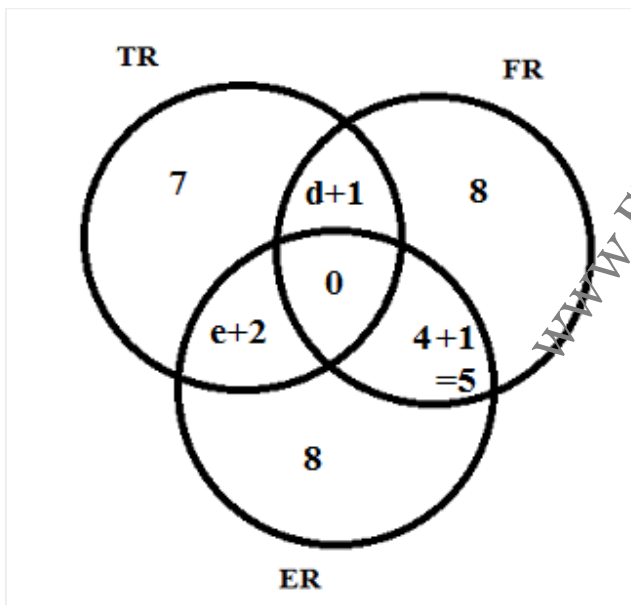
It was given that total number of volunteers are 37. But total how many projects they are volunteering?



$$\text{Total projects} = (7 + 8 + 8) \times 1 + (d + e + 4) \times 2 + 4 \times 3 = 23 + 20 + 12 = 55$$

In the above discussion, we have seen that volunteers are withdrawing from projects. In that case what happens to the number of projects and volunteers?

Volunteers remain same. i.e., 37. But number of projects came down.



Let us calculate the number of projects in this case

$$(7 + 8 + 8) \times 1 + (d + 1 + e + 2 + 5) \times 2 = 23 + 28 = 51.$$

There are 4 projects less than earlier. This is because, 4 volunteers withdrawn from working on 3 projects to 2 projects. That is why total projects got reduced by exactly 4. Also number of volunteers in exactly two projects also go up by 4.

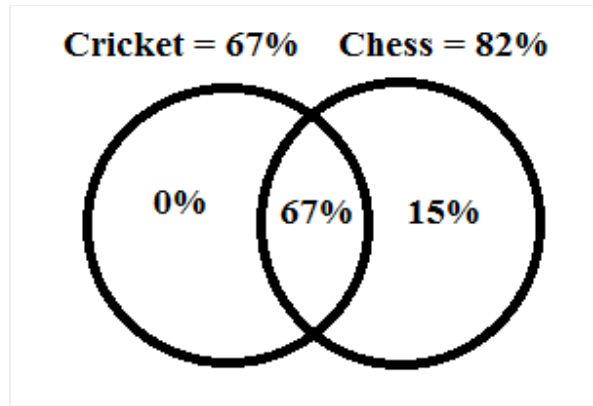
[Concept of Maxima and Minima in sets:](#)

In a class, 67% play cricket, and 82% play chess.

Question 1: What is the maximum number of students who can play cricket as well as chess.

Question 2: What is the minimum number of students who can play cricket as well as chess, given that each students play atleast one game.

Ans 1: If all the students who play cricket also play chess... then

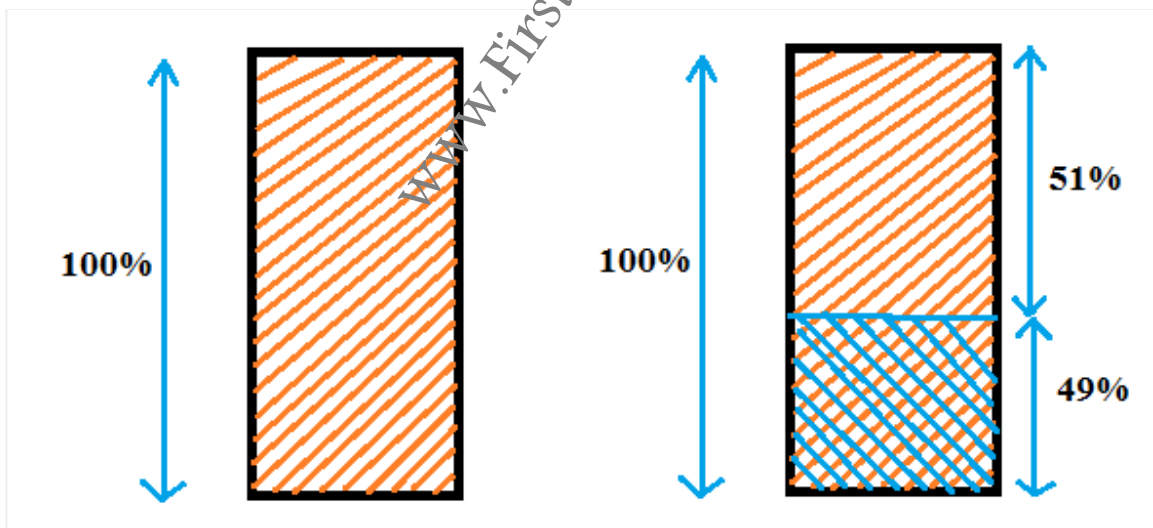


Total number of students who play atleast one game = $0 + 67 + 15 = 82$

So students who does not play any game = $100 - 82 = 18$.

Ans 2: Total games are $67 + 82 = 149$.

These 149 games are played by 100 students. As each student play atleast one game, there must be some who play both the games.



Give one game to all 100 students. The we have excess of $149 - 100 = 49$ games.

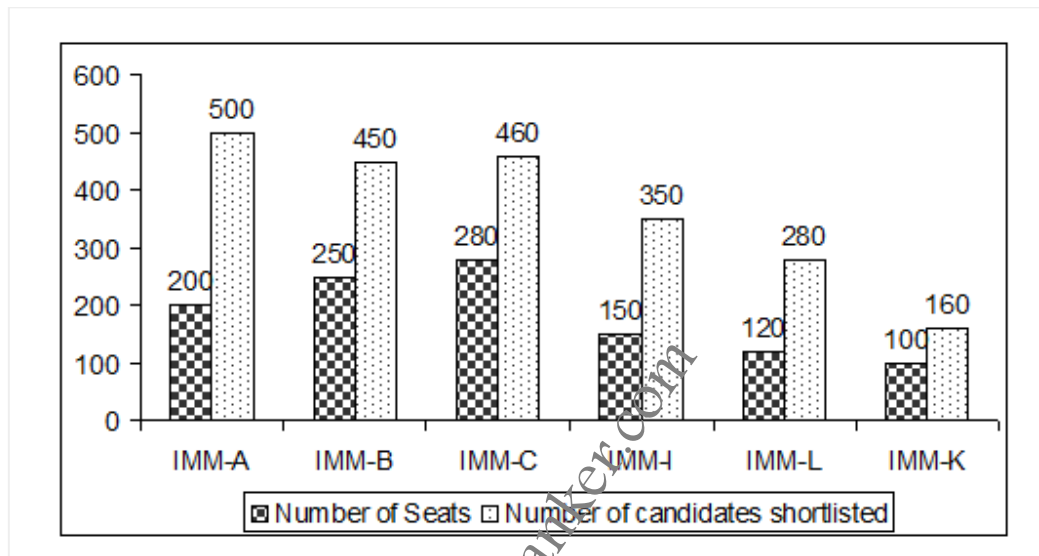
Now we distribute these 49 to 49 students who have already got one game. So there are minimum 49 students who can play 2 games, and the remaining $100 - 49 = 51$ play one game.

Observe total games are $49 \times 2 + 51 = 149$ remains same.

There are six institutes IMM-A, IMM-B, IMM-C, IMM-I, IMM-L, and IMM-K in a country named Crazyland. Every year these six institutes conduct a Common Intelligence Test (CIT) and send interview calls to students for the total of 1100 seats distributed among the six IMMs. After the interview process is over, each IMM releases the Final List

giving the names of the candidates finally offered a seat and Wait List giving the names of the candidates selected in the waiting list in case the candidates in the Final List chose not to accept the offer of the given IMM. A given candidate may get shortlisted in the Final List chose not to accept the offer of the given IMM. Finally it was found that 1500 students received calls from one or more than one IMM. As some students received multiple calls, the sum of total number of calls sent out by individual IMMs exceeded the total number of students receiving calls.

The following Bar graph gives the number of seats and the number of candidates shortlisted either in the Final List or in the Wait List of each of the six IMMs



1. What can be the maximum number of candidates whose name appeared either in the Final list or the Wait List of all the six IMMs?

Ans: Given that, the number of students who received atleast one call = 1500

Total calls given = $500 + 450 + 460 + 350 + 280 + 160 = 2200$

So give one call to each of these 1500 students, then we have $2200 - 1500 = 700$ calls surplus.

We know that a student can get a maximum of 6 calls. So Some of these 1500 students may get a maximum of 5 calls more. So $700 / 5 = 140$ students may get another 5 calls. So there are 140 students who can get 6 calls.

2. What is the maximum number of students who received multiple calls?

Ans: From the above observation, we know that there are 700 calls surplus to be distributed. We consider getting 2 calls also a multiple call. To maximise the multiple call holders, we give these 700 calls to 700 students. So there are 700 students who got 2 calls, remaining 800 got one call.

3. If it is known that 100 students got selected in either Final or Wait list of exactly 5 IMMs, then what is the maximum number of students who can get selected in exactly 4 IMMs?

Ans: If 100 students got 5 calls they have a total of 500 calls with them. So the remaining 1400 students must have got $2200 - 500 = 1700$ calls.

Giving one call each to these 1400 students, we have 300 calls surplus. To maximize the number of students who got exactly 4 calls, we can give another 3 calls to some of these 1400 students. $300 / 3 = 100$.

So maximum number of students who got exactly 4 calls = 100

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