

Notations :

- 1.Options shown in green color and with ✓ icon are correct.
- 2.Options shown in red color and with ✗ icon are incorrect.

Question Paper Name: MA: MATHEMATICS 1st Feb shift2  
Number of Questions: 65  
Total Marks: 100.0

Wrong answer for MCQ will result in negative marks, (-1/3) for 1 mark Questions and (-2/3) for 2 marks Questions.

General Aptitude

Number of Questions: 10  
Section Marks: 15.0

Q.1 to Q.5 carry 1 mark each & Q.6 to Q.10 carry 2 marks each.

Question Number : 1 Question Type : MCQ

Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:

Apparent lifelessness \_\_\_\_\_ dormant life.  
(A) harbours (B) leads to (C) supports (D) affects

Options :

- 1. ✓ A
- 2. ✗ B
- 3. ✗ C
- 4. ✗ D

Question Number : 2 Question Type : MCQ

Fill in the blank with the correct idiom/phrase.

That boy from the town was a \_\_\_\_\_ in the sleepy village.  
(A) dog out of herd (B) sheep from the heap  
(C) fish out of water (D) bird from the flock

Options :

- 1. ✗ A
- 2. ✗ B
- 3. ✓ C
- 4. ✗ D

Question Number : 3 Question Type : MCQ

Choose the sentence in which the underlined word is used correctly.

- (A) When the teacher eludes to different authors, he is being elusive.
- (B) When the thief keeps eluding the police, he is being elusive.
- (C) Matters that are difficult to understand, identify or remember are allusive.
- (D) Mirages can be allusive, but a better way to express them is illusory.

Options :

- 1. ✖ A
- 2. ✔ B
- 3. ✖ C
- 4. ✖ D

Question Number : 4 Question Type : MCQ

Tanya is older than Eric.  
Cliff is older than Tanya.  
Eric is older than Cliff.

If the first two statements are true, then the third statement is:

- (A) True
- (B) False
- (C) Uncertain
- (D) Data insufficient

Options :

- 1. ✖ A
- 2. ✔ B
- 3. ✖ C
- 4. ✖ D

Question Number : 5 Question Type : MCQ

Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?

- (A) 20                      (B) 10                      (C) 8                      (D) 5

Options :

- 1. ✖ A
- 2. ✔ B
- 3. ✖ C
- 4. ✖ D

Question Number : 6 Question Type : MCQ

Increased productivity necessary reflects greater efforts made by the employees.

- (A) Increase in productivity necessary
- (B) Increase productivity is necessary
- (C) Increase in productivity necessarily
- (D) No improvement required

Options :

- 1. ✗ A
- 2. ✗ B
- 3. ✓ C
- 4. ✗ D

Question Number : 7 Question Type : MCQ

Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:

- I. No manager is a leader.
- II. All leaders are executives.

Conclusions:

- I. No manager is an executive.
- II. No executive is a manager.

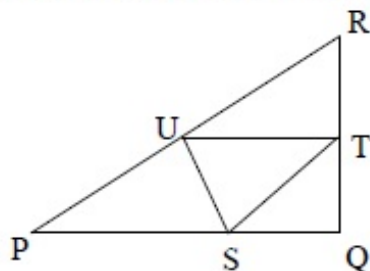
- (A) Only conclusion I follows.
- (B) Only conclusion II follows.
- (C) Neither conclusion I nor II follows.
- (D) Both conclusions I and II follow.

Options :

- 1. ✗ A
- 2. ✗ B
- 3. ✓ C
- 4. ✗ D

Question Number : 8 Question Type : NAT

In the given figure angle Q is a right angle,  $PS:QS = 3:1$ ,  $RT:QT = 5:2$  and  $PU:UR = 1:1$ . If area of triangle QTS is  $20 \text{ cm}^2$ , then the area of triangle PQR in  $\text{cm}^2$  is \_\_\_\_\_.



Correct Answer :

280

Right triangle PQR is to be constructed in the  $xy$  - plane so that the right angle is at P and line PR is parallel to the  $x$ -axis. The  $x$  and  $y$  coordinates of P, Q, and R are to be integers that satisfy the inequalities:  $-4 \leq x \leq 5$  and  $6 \leq y \leq 16$ . How many different triangles could be constructed with these properties?

- (A) 110 (B) 1,100 (C) 9,900 (D) 10,000

Options :

1. ✖ A  
2. ✖ B  
3. ✔ C  
4. ✖ D

Question Number : 10 Question Type : MCQ

A coin is tossed thrice. Let  $X$  be the event that head occurs in each of the first two tosses. Let  $Y$  be the event that a tail occurs on the third toss. Let  $Z$  be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

- (A)  $X$  and  $Y$  are not independent (B)  $Y$  and  $Z$  are dependent  
(C)  $Y$  and  $Z$  are independent (D)  $X$  and  $Z$  are independent

Options :

1. ✖ A  
2. ✔ B  
3. ✖ C  
4. ✖ D

Mathematics

Number of Questions:

55

Section Marks:

85.0

Q.11 to Q.35 carry 1 mark each & Q.36 to Q.65 carry 2 marks each.

Question Number : 11 Question Type : NAT

Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  be a linear map defined by

$$T(x, y, z, w) = (x + z, 2x + y + 3z, 2y + 2z, w).$$

Then the rank of  $T$  is equal to \_\_\_\_\_

Correct Answer :

3

Question Number : 12 Question Type : NAT

Let  $M$  be a  $3 \times 3$  matrix and suppose that 1, 2 and 3 are the eigenvalues of  $M$ . If

$$M^{-1} = \frac{M^2}{\alpha} - M + \frac{11}{\alpha} I_3$$

for some scalar  $\alpha \neq 0$ , then  $\alpha$  is equal to \_\_\_\_\_

Correct Answer :

6

Question Number : 13 Question Type : NAT

Let  $M$  be a  $3 \times 3$  singular matrix and suppose that 2 and 3 are eigenvalues of  $M$ . Then the number of linearly independent eigenvectors of  $M^3 + 2M + I_3$  is equal to \_\_\_\_\_

Correct Answer :

3

Question Number : 14 Question Type : NAT

Let  $M$  be a  $3 \times 3$  matrix such that  $M \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$  and suppose that  $M^3 \begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$  for some  $\alpha, \beta, \gamma \in \mathbb{R}$ . Then  $|\alpha|$  is equal to \_\_\_\_\_

Correct Answer:

27

Question Number : 15 Question Type : MCQ

Let  $f: [0, \infty) \rightarrow \mathbb{R}$  be defined by

$$f(x) = \int_0^x \sin^2(t^2) dt.$$

Then the function  $f$  is

- (A) uniformly continuous on  $[0, 1)$  but NOT on  $(0, \infty)$
- (B) uniformly continuous on  $(0, \infty)$  but NOT on  $[0, 1)$
- (C) uniformly continuous on both  $[0, 1)$  and  $(0, \infty)$
- (D) neither uniformly continuous on  $[0, 1)$  nor uniformly continuous on  $(0, \infty)$

Options :

- 1. ✖ A
- 2. ✖ B
- 3. ✔ C
- 4. ✖ D

Question Number : 16 Question Type : NAT



Consider the power series  $\sum_{n=0}^{\infty} a_n z^n$ , where  $a_n = \begin{cases} \frac{1}{3^n} & \text{if } n \text{ is even} \\ \frac{1}{5^n} & \text{if } n \text{ is odd.} \end{cases}$

The radius of convergence of the series is equal to \_\_\_\_\_

Correct Answer :

3

Question Number : 17 Question Type : NAT

Let  $C = \{ z \in \mathbb{C} : |z - i| = 2 \}$ . Then  $\frac{1}{2\pi} \oint_C \frac{z^2 - 4}{z^2 + 4} dz$  is equal to \_\_\_\_\_

Correct Answer :

-2

Question Number : 18 Question Type : NAT

Let  $X \sim B(5, \frac{1}{2})$  and  $Y \sim U(0,1)$ . Then  $\frac{P(X+Y \leq 2)}{P(X+Y \geq 5)}$  is equal to \_\_\_\_\_

Correct Answer:

6

Question Number : 19 Question Type : NAT

Let the random variable  $X$  have the distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{2} & \text{if } 0 \leq x < 1 \\ \frac{3}{5} & \text{if } 1 \leq x < 2 \\ \frac{1}{2} + \frac{x}{8} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3. \end{cases}$$

Then  $P(2 \leq X < 4)$  is equal to \_\_\_\_\_

Correct Answer:

0.4

Question Number : 20 Question Type : NAT

Let  $X$  be a random variable having the distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \leq x < 1 \\ \frac{1}{3} & \text{if } 1 \leq x < 2 \\ \frac{1}{2} & \text{if } 2 \leq x < \frac{11}{3} \\ 1 & \text{if } x \geq \frac{11}{3}. \end{cases}$$

Then  $E(X)$  is equal to \_\_\_\_\_

Correct Answer :

2.25

Question Number : 21 Question Type : MCQ

In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to

- (A)  $\frac{125}{6^5}$  (B)  $\frac{150}{6^5}$  (C)  $\frac{175}{6^5}$  (D)  $\frac{200}{6^5}$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 22 Question Type : MCQ

Let  $x_1 = 2.2$ ,  $x_2 = 4.3$ ,  $x_3 = 3.1$ ,  $x_4 = 4.5$ ,  $x_5 = 1.1$  and  $x_6 = 5.7$  be the observed values of a random sample of size 6 from a  $U(\theta - 1, \theta + 4)$  distribution, where  $\theta \in (0, \infty)$  is unknown. Then a maximum likelihood estimate of  $\theta$  is equal to

- (A) 1.8 (B) 2.3 (C) 3.1 (D) 3.6

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 23 Question Type : MCQ

Let  $\Omega = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$  be the open unit disc in  $\mathbb{R}^2$  with boundary  $\partial\Omega$ . If  $u(x, y)$  is the solution of the Dirichlet problem

$$\begin{aligned} u_{xx} + u_{yy} &= 0 && \text{in } \Omega \\ u(x, y) &= 1 - 2y^2 && \text{on } \partial\Omega, \end{aligned}$$

then  $u\left(\frac{1}{2}, 0\right)$  is equal to

- (A)  $-1$  (B)  $\frac{-1}{4}$  (C)  $\frac{1}{4}$  (D)  $1$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 24 Question Type : NAT

Let  $c \in \mathbb{Z}_3$  be such that  $\frac{\mathbb{Z}_3[X]}{\langle X^2 + cX + 1 \rangle}$  is a field. Then  $c$  is equal to \_\_\_\_\_

Correct Answer :

2

Question Number : 25 Question Type : MCQ

Let  $V = C^1[0, 1]$ ,  $X = (C[0, 1], \|\cdot\|_\infty)$  and  $Y = (C[0, 1], \|\cdot\|_2)$ . Then  $V$  is

- (A) dense in  $X$  but NOT in  $Y$   
 (B) dense in  $Y$  but NOT in  $X$   
 (C) dense in both  $X$  and  $Y$   
 (D) neither dense in  $X$  nor dense in  $Y$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 26 Question Type : NAT

Let  $T : (C[0, 1], \|\cdot\|_\infty) \rightarrow \mathbb{R}$  be defined by  $T(f) = \int_0^1 2xf(x) dx$  for all  $f \in C[0, 1]$ . Then  $\|T\|$  is equal to \_\_\_\_\_

Correct Answer :

1

Question Number : 27 Question Type : MCQ



Let  $\tau_1$  be the usual topology on  $\mathbb{R}$ . Let  $\tau_2$  be the topology on  $\mathbb{R}$  generated by

$\mathcal{B} = \{[a, b) \subset \mathbb{R} : -\infty < a < b < \infty\}$ . Then the set  $\{x \in \mathbb{R} : 4 \sin^2 x \leq 1\} \cup \left\{\frac{\pi}{2}\right\}$  is

- (A) closed in  $(\mathbb{R}, \tau_1)$  but NOT in  $(\mathbb{R}, \tau_2)$
- (B) closed in  $(\mathbb{R}, \tau_2)$  but NOT in  $(\mathbb{R}, \tau_1)$
- (C) closed in both  $(\mathbb{R}, \tau_1)$  and  $(\mathbb{R}, \tau_2)$
- (D) neither closed in  $(\mathbb{R}, \tau_1)$  nor closed in  $(\mathbb{R}, \tau_2)$

Options :

- 1. ✖ A
- 2. ✖ B
- 3. ✔ C
- 4. ✖ D

Question Number : 28 Question Type : MCQ

Let  $X$  be a connected topological space such that there exists a non-constant continuous function  $f : X \rightarrow \mathbb{R}$ , where  $\mathbb{R}$  is equipped with the usual topology. Let  $f(X) = \{f(x) : x \in X\}$ . Then

- (A)  $X$  is countable but  $f(X)$  is uncountable
- (B)  $f(X)$  is countable but  $X$  is uncountable
- (C) both  $f(X)$  and  $X$  are countable
- (D) both  $f(X)$  and  $X$  are uncountable

Options :

- 1. ✖ A
- 2. ✖ B
- 3. ✖ C
- 4. ✔ D

Question Number : 29 Question Type : MCQ

Let  $d_1$  and  $d_2$  denote the usual metric and the discrete metric on  $\mathbb{R}$ , respectively.

Let  $f : (\mathbb{R}, d_1) \rightarrow (\mathbb{R}, d_2)$  be defined by  $f(x) = x$ ,  $x \in \mathbb{R}$ . Then

- (A)  $f$  is continuous but  $f^{-1}$  is NOT continuous
- (B)  $f^{-1}$  is continuous but  $f$  is NOT continuous
- (C) both  $f$  and  $f^{-1}$  are continuous
- (D) neither  $f$  nor  $f^{-1}$  is continuous

Options :

- 1. ✖ A
- 2. ✔ B
- 3. ✖ C
- 4. ✖ D

Question Number : 30 Question Type : NAT

If the trapezoidal rule with single interval  $[0, 1]$  is exact for approximating the integral

$\int_0^1 (x^3 - c x^2) dx$ , then the value of  $c$  is equal to \_\_\_\_\_

Question Number : 31 Question Type : NAT

Suppose that the Newton-Raphson method is applied to the equation  $2x^2 + 1 - e^{x^2} = 0$  with an initial approximation  $x_0$  sufficiently close to zero. Then, for the root  $x = 0$ , the order of convergence of the method is equal to \_\_\_\_\_

Correct Answer :

1

Question Number : 32 Question Type : NAT

The minimum possible order of a homogeneous linear ordinary differential equation with real constant coefficients having  $x^2 \sin(x)$  as a solution is equal to \_\_\_\_\_

Correct Answer:

6

Question Number : 33 Question Type : MCQ

The Lagrangian of a system in terms of polar coordinates  $(r, \theta)$  is given by

$$L = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m (\dot{\theta}^2 + r^2 \dot{\theta}^2) - m g r (1 - \cos(\theta)),$$

where  $m$  is the mass,  $g$  is the acceleration due to gravity and  $\dot{s}$  denotes the derivative of  $s$  with respect to time. Then the equations of motion are

(A)  $2 \ddot{r} = r \dot{\theta}^2 - g (1 - \cos(\theta)), \quad \frac{d}{dt}(r^2 \dot{\theta}) = -g r \sin(\theta)$

(B)  $2 \ddot{r} = r \dot{\theta}^2 + g (1 - \cos(\theta)), \quad \frac{d}{dt}(r^2 \dot{\theta}) = -g r \sin(\theta)$

(C)  $2 \ddot{r} = r \dot{\theta}^2 - g (1 - \cos(\theta)), \quad \frac{d}{dt}(r^2 \dot{\theta}) = g r \sin(\theta)$

(D)  $2 \ddot{r} = r \dot{\theta}^2 + g (1 - \cos(\theta)), \quad \frac{d}{dt}(r^2 \dot{\theta}) = g r \sin(\theta)$

Options :

1. ✓ A

2. ✗ B

3. ✗ C

4. ✗ D

Question Number : 34 Question Type : NAT

If  $y(x)$  satisfies the initial value problem

$$(x^2 + y)dx = x dy, \quad y(1) = 2,$$

then  $y(2)$  is equal to \_\_\_\_\_

Question Number : 35 Question Type : NAT

It is known that Bessel functions  $J_n(x)$ , for  $n \geq 0$ , satisfy the identity

$$e^{\frac{x}{2}\left(t - \frac{1}{t}\right)} = J_0(x) + \sum_{n=1}^{\infty} J_n(x) \left( t^n + \frac{(-1)^n}{t^n} \right)$$

for all  $t > 0$  and  $x \in \mathbb{R}$ . The value of  $J_0\left(\frac{\pi}{3}\right) + 2 \sum_{n=1}^{\infty} J_{2n}\left(\frac{\pi}{3}\right)$  is equal to \_\_\_\_\_

Correct Answer :

Question Number : 36 Question Type : MCQ

Let  $X$  and  $Y$  be two random variables having the joint probability density function

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then the conditional probability  $P\left(X \leq \frac{2}{3} \mid Y = \frac{3}{4}\right)$  is equal to

- (A)  $\frac{5}{9}$  (B)  $\frac{2}{3}$  (C)  $\frac{7}{9}$  (D)  $\frac{8}{9}$

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 37 Question Type : NAT

Let  $\Omega = (0, 1]$  be the sample space and let  $P(\cdot)$  be a probability function defined by

$$P((0, x]) = \begin{cases} \frac{x}{2} & \text{if } 0 \leq x < \frac{1}{2} \\ x & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then  $P\left(\left\{\frac{1}{2}\right\}\right)$  is equal to \_\_\_\_\_

Correct Answer :

Question Number : 38 Question Type : NAT

www.FirstRanker.com

Let  $X_1, X_2$  and  $X_3$  be independent and identically distributed random variables with  $E(X_1) = 0$  and  $E(X_1^2) = \frac{15}{4}$ . If  $\psi : (0, \infty) \rightarrow (0, \infty)$  is defined through the conditional expectation

$$\psi(t) = E(X_1^2 \mid X_1^2 + X_2^2 + X_3^2 = t), \quad t > 0,$$

then  $E(\psi((X_1 + X_2)^2))$  is equal to \_\_\_\_\_

Correct Answer :

2.5

Question Number : 39 Question Type : NAT

Let  $X \sim \text{Poisson}(\lambda)$ , where  $\lambda > 0$  is unknown. If  $\delta(X)$  is the unbiased estimator of  $g(\lambda) = e^{-\lambda}(3\lambda^2 + 2\lambda + 1)$ , then  $\sum_{k=0}^{\infty} \delta(k)$  is equal to \_\_\_\_\_

Correct Answer :

9

Question Number : 40 Question Type : NAT

Let  $X_1, \dots, X_n$  be a random sample from  $N(\mu, 1)$  distribution, where  $\mu \in \{0, \frac{1}{2}\}$ . For testing the null hypothesis  $H_0: \mu = 0$  against the alternative hypothesis  $H_1: \mu = \frac{1}{2}$ , consider the critical region

$$R = \left\{ (x_1, x_2, \dots, x_n) : \sum_{i=1}^n x_i > c \right\},$$

where  $c$  is some real constant. If the critical region  $R$  has size 0.025 and power 0.7054, then the value of the sample size  $n$  is equal to \_\_\_\_\_

Correct Answer :

25

Question Number : 41 Question Type : MCQ

Let  $X$  and  $Y$  be independently distributed central chi-squared random variables with degrees of freedom  $m (\geq 3)$  and  $n (\geq 3)$ , respectively. If  $E\left(\frac{X}{Y}\right) = 3$  and  $m + n = 14$ , then  $E\left(\frac{Y}{X}\right)$  is equal to

- (A)  $\frac{2}{7}$                       (B)  $\frac{3}{7}$                       (C)  $\frac{4}{7}$                       (D)  $\frac{5}{7}$

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D



Question Number : 42 Question Type : NAT

Let  $X_1, X_2, \dots$  be a sequence of independent and identically distributed random variables with  $P(X_1 = 1) = \frac{1}{4}$  and  $P(X_1 = 2) = \frac{3}{4}$ . If  $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$ , for  $n = 1, 2, \dots$ , then  $\lim_{n \rightarrow \infty} P(\bar{X}_n \leq 1.8)$  is equal to \_\_\_\_\_

Correct Answer:

1

Question Number : 43 Question Type : MCQ

Let  $u(x, y) = 2f(y) \cos(x - 2y)$ ,  $(x, y) \in \mathbb{R}^2$ , be a solution of the initial value problem

$$\begin{aligned} 2u_x + u_y &= u \\ u(x, 0) &= \cos(x). \end{aligned}$$

Then  $f(1)$  is equal to

- (A)  $\frac{1}{2}$  (B)  $\frac{e}{2}$  (C)  $e$  (D)  $\frac{3e}{2}$

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Question Number : 44 Question Type : NAT

Let  $u(x, t)$ ,  $x \in \mathbb{R}$ ,  $t \geq 0$ , be the solution of the initial value problem

$$\begin{aligned} u_{tt} &= u_{xx} \\ u(x, 0) &= x \\ u_t(x, 0) &= 1. \end{aligned}$$

Then  $u(2, 2)$  is equal to \_\_\_\_\_

Correct Answer:

4

Question Number : 45 Question Type : NAT

Let  $W = \text{Span} \left\{ \frac{1}{\sqrt{2}}(0, 0, 1, 1), \frac{1}{\sqrt{2}}(1, -1, 0, 0) \right\}$  be a subspace of the Euclidean space  $\mathbb{R}^4$ . Then the square of the distance from the point  $(1, 1, 1, 1)$  to the subspace  $W$  is equal to \_\_\_\_\_

Correct Answer :

2



Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  be a linear map such that the null space of  $T$  is

$$\{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0\}$$

and the rank of  $(T - 4I_4)$  is 3. If the minimal polynomial of  $T$  is  $x(x - 4)^\alpha$ , then  $\alpha$  is equal to \_\_\_\_\_

Correct Answer :

1

Question Number : 47 Question Type : MCQ

Let  $M$  be an invertible Hermitian matrix and let  $x, y \in \mathbb{R}$  be such that  $x^2 < 4y$ . Then

- (A) both  $M^2 + xM + yI$  and  $M^2 - xM + yI$  are singular
- (B)  $M^2 + xM + yI$  is singular but  $M^2 - xM + yI$  is non-singular
- (C)  $M^2 + xM + yI$  is non-singular but  $M^2 - xM + yI$  is singular
- (D) both  $M^2 + xM + yI$  and  $M^2 - xM + yI$  are non-singular

Options :

- 1. ✖ A
- 2. ✖ B
- 3. ✖ C
- 4. ✔ D

Question Number : 48 Question Type : MCQ

Let  $G = \{e, x, x^2, x^3, y, xy, x^2y, x^3y\}$  with  $o(x) = 4, o(y) = 2$  and  $xy = yx^3$ . Then the number of elements in the center of the group  $G$  is equal to

- (A) 1
- (B) 2
- (C) 4
- (D) 8

Options :

- 1. ✖ A
- 2. ✔ B
- 3. ✖ C
- 4. ✖ D

Question Number : 49 Question Type : NAT

The number of ring homomorphisms from  $\mathbb{Z}_2 \times \mathbb{Z}_2$  to  $\mathbb{Z}_4$  is equal to \_\_\_\_\_

Correct Answer:

1

Question Number : 50 Question Type : MCQ

Let  $p(x) = x^5 + 10x^3 + 5x + 15$  and  $q(x) = x^3 - x^2 - x - 2$  be two polynomials in  $\mathbb{Q}[x]$ . Then, over  $\mathbb{Q}$ ,

- (A)  $p(x)$  and  $q(x)$  are both irreducible
- (B)  $p(x)$  is reducible but  $q(x)$  is irreducible
- (C)  $p(x)$  is irreducible but  $q(x)$  is reducible
- (D)  $p(x)$  and  $q(x)$  are both reducible

Options :

- 1. ✖ A
- 2. ✖ B
- 3. ✔ C
- 4. ✖ D

Question Number : 51 Question Type : NAT

Consider the linear programming problem

$$\begin{aligned} &\text{Maximize } 3x + 9y, \\ &\text{subject to } \begin{aligned} 2y - x &\leq 2 \\ 3y - x &\geq 0 \\ 2x + 3y &\leq 10 \\ x, y &\geq 0. \end{aligned} \end{aligned}$$

Then the maximum value of the objective function is equal to \_\_\_\_\_

Correct Answer:

24

Question Number : 52 Question Type : MCQ

Let  $S = \{ (x, \sin \frac{1}{x}) : 0 < x \leq 1 \}$  and  $T = S \cup \{(0,0)\}$ . Under the usual metric on  $\mathbb{R}^2$ ,

- (A)  $S$  is closed but  $T$  is NOT closed
- (B)  $T$  is closed but  $S$  is NOT closed
- (C) both  $S$  and  $T$  are closed
- (D) neither  $S$  nor  $T$  is closed

Options :

- 1. ✖ A
- 2. ✖ B
- 3. ✖ C
- 4. ✔ D

Question Number : 53 Question Type : MCQ

Let  $H = \{ (x_n) \in \ell_2 : \sum_{n=1}^{\infty} \frac{x_n}{n} = 1 \}$ . Then  $H$

- (A) is bounded
- (B) is closed
- (C) is a subspace
- (D) has an interior point

Options :

- 1. ✖ A
- 2. ✔ B

Question Number : 54 Question Type : MCQ

Let  $V$  be a closed subspace of  $L^2[0, 1]$  and let  $f, g \in L^2[0, 1]$  be given by  $f(x) = x$  and  $g(x) = x^2$ . If  $V^\perp = \text{Span}\{f\}$  and  $Pg$  is the orthogonal projection of  $g$  on  $V$ , then  $(g - Pg)(x)$ ,  $x \in [0, 1]$ , is

- (A)  $\frac{3}{4}x$  (B)  $\frac{1}{4}x$  (C)  $\frac{3}{4}x^2$  (D)  $\frac{1}{4}x^2$

Options :

1. ✔ A  
2. ✖ B  
3. ✖ C  
4. ✖ D

Question Number : 55 Question Type : NAT

Let  $p(x)$  be the polynomial of degree at most 3 that passes through the points  $(-2, 12)$ ,  $(-1, 1)$ ,  $(0, 2)$  and  $(2, -8)$ . Then the coefficient of  $x^3$  in  $p(x)$  is equal to \_\_\_\_\_

Correct Answer :

-2

Question Number : 56 Question Type : NAT

If, for some  $\alpha, \beta \in \mathbb{R}$ , the integration formula

$$\int_0^2 p(x) dx = p(\alpha) + p(\beta)$$

holds for all polynomials  $p(x)$  of degree at most 3, then the value of  $3(\alpha - \beta)^2$  is equal to \_\_\_\_\_

Correct Answer :

4

Question Number : 57 Question Type : NAT

Let  $y(t)$  be a continuous function on  $[0, \infty)$  whose Laplace transform exists. If  $y(t)$  satisfies

$$\int_0^t (1 - \cos(t - \tau)) y(\tau) d\tau = t^4,$$

then  $y(1)$  is equal to \_\_\_\_\_

Correct Answer :

28

Consider the initial value problem

$$x^2 y'' - 6y = 0, \quad y(1) = \alpha, \quad y'(1) = 6.$$

If  $y(x) \rightarrow 0$  as  $x \rightarrow 0^+$ , then  $\alpha$  is equal to \_\_\_\_\_

Correct Answer :

2

Question Number : 59 Question Type : MCQ

Define  $f_1, f_2: [0, 1] \rightarrow \mathbb{R}$  by

$$f_1(x) = \sum_{n=1}^{\infty} \frac{x \sin(n^2 x)}{n^2} \quad \text{and} \quad f_2(x) = \sum_{n=1}^{\infty} x^2 (1 - x^2)^{n-1}.$$

Then

- (A)  $f_1$  is continuous but  $f_2$  is NOT continuous
- (B)  $f_2$  is continuous but  $f_1$  is NOT continuous
- (C) both  $f_1$  and  $f_2$  are continuous
- (D) neither  $f_1$  nor  $f_2$  is continuous

Options :

- 1. ✓ A
- 2. ✗ B
- 3. ✗ C
- 4. ✗ D

Question Number : 60 Question Type : NAT

Consider the unit sphere  $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$  and the unit normal vector  $\hat{n} = (x, y, z)$  at each point  $(x, y, z)$  on  $S$ . The value of the surface integral

$$\iint_S \left\{ \left( \frac{2x}{\pi} + \sin(y^2) \right) x + \left( e^z - \frac{y}{\pi} \right) y + \left( \frac{2z}{\pi} + \sin^2 y \right) z \right\} d\sigma$$

is equal to \_\_\_\_\_

Correct Answer :

4

Question Number : 61 Question Type : NAT

Let  $D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq 1000, 1 \leq y \leq 1000\}$ . Define

$$f(x, y) = \frac{xy}{2} + \frac{500}{x} + \frac{500}{y}.$$

Then the minimum value of  $f$  on  $D$  is equal to \_\_\_\_\_

Correct Answer:

150

Question Number : 62 Question Type : MCQ

Let  $\mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \}$ . Then there exists a non-constant analytic function  $f$  on  $\mathbb{D}$  such that for all  $n = 2, 3, 4, \dots$

- (A)  $f\left(\frac{\sqrt{-1}}{n}\right) = 0$
- (B)  $f\left(\frac{1}{n}\right) = 0$
- (C)  $f\left(1 - \frac{1}{n}\right) = 0$
- (D)  $f\left(\frac{1}{2} - \frac{1}{n}\right) = 0$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 63 Question Type : NAT

Let  $\sum_{n=-\infty}^{\infty} a_n z^n$  be the Laurent series expansion of  $f(z) = \frac{1}{2z^2 - 13z + 15}$  in the annulus  $\frac{3}{2} < |z| < 5$ . Then  $\frac{a_1}{a_2}$  is equal to \_\_\_\_\_

Correct Answer:

5

Question Number : 64 Question Type : NAT

The value of  $\frac{i}{4-\pi} \int_{|z|=4} \frac{dz}{z \cos(z)}$  is equal to \_\_\_\_\_

Correct Answer :

2

Question Number : 65 Question Type : MCQ

Suppose that among all continuously differentiable functions  $y(x)$ ,  $x \in \mathbb{R}$ , with  $y(0) = 0$  and  $y(1) = \frac{1}{2}$ , the function  $y_0(x)$  minimizes the functional

$$\int_0^1 (e^{-(y'-x)} + (1+y)y') dx.$$

Then  $y_0\left(\frac{1}{2}\right)$  is equal to

- (A) 0
- (B)  $\frac{1}{8}$
- (C)  $\frac{1}{4}$
- (D)  $\frac{1}{2}$

Options :

1. ✖ A



3. ✖ C

4. ✖ D