Code No: R5100102 **R5**

B.Tech I Year(R05) Supplementary Examinations, May/June 2010
MATHEMATICS-I
(Common to all branches)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sum n! \,\frac{2^n}{n^n}$ [5]
   (b) State and prove Cauchy Mean value theorem. [5]
   (c) If $0 < a < b < 1$, using Lagrange's mean value theorem, prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}$ [6]
2. (a) Find Taylor's expansion of f(x,y) = $\cot^{-1}$xy in powers of (x+0.5) and (y-2) up to second degree terms.
   (b) Show that the evolute of $x = a\left(cos\theta + \log\tan\frac{\theta}{2}\right)$, y = a sin$\theta$ is the catenary $y = a\,cosh\frac{x}{a}$. [8+8]
3. (a) Find the surface area of the solid generated by the revolution of the loop of the curve $x = t^2$, $y = t - \frac{t^3}{3}$ about x aixs.
   (b) Find the area of the loop of the curve $y^2(a + x) = x^2(x - a)$. [8+8]
4. (a) Form the differential equations of all circles which passes through the origin whose centres lie on x axis.
   (b) Solve the differential equation $\frac{dy}{dx} + yx = y^2 e^{x^2/2}\sin x$.
   (c) Find the orthogonal Trajectories of the family of curves r=2a(cosθ+sinθ). [4+6+6]
5. (a) Solve the differential equation: $(D^3 + 1)y = \sin 3x - \cos^2 x$.
   (b) Solve the differential equation: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1 + x)^2$. [6+10]
6. (a) Find the Laplace Transformations of the following functions $e^{-3t}(2\cos 5t - 3\sin 5t)$
   (b) Find $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$
   (c) Evaluate: $\int\limits_0^1 \int\limits_0^{\sqrt{1+x^2}} \frac{dx\,dy}{(1+x^2+y^2)}$ [5+6+5]
7. (a) Prove that div(**A**×**B**)=**B**.curl**A** - **A**.curl**B**.
   (b) Find the directional derivative of the scalar point function ϕ (x,y,z) = $4xy^2 + 2x^2yz$ at the point A(1, 2, 3) in the direction of the line AB where B = (5,0,4). [8+8]
8. State Green's theorem and verify Green's theorem for $\oint\limits_C [(xy + y^2)dx + x^2dy]$, where C is bounded by y = x and y = x^2. [16]

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