

B.Tech. I Year(RR) Supplementary Examinations, May/June 2010
MATHEMATICS-I
 (Common to all branches)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the convergence of the series $\sum \frac{(n!)^2}{(2n)!} x^{2n}$ [5]
 (b) Test the following series for absolute /conditional convergence

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$$
 [6]
 (c) Show that $\sin^{-1} x = x + \frac{x^3}{3!} + \frac{1^2 \cdot 3^2}{5!} x^5 + \frac{1^2 \cdot 3^2 \cdot 5^2}{7!} x^7 + \dots$ [5]
2. (a) Find the stationary points of the following function 'u' and find the maximum or the minimum
 $u = x^2 + 2xy + 2y^2 + 2x + y$
 (b) Considering the evolute of a curve as the envelope of its normals, find the evolute of the ellipse
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [8+8]
3. (a) Trace the Folium of Decartes : $x^3 + y^3 = 3axy$.
 (b) Determine the volume of the solid generated by revolving the limaçon
 $r = a + b \cos \theta$ ($a > b$) about the initial line. [8+8]
4. (a) Obtain the differential equation of the coaxial circles of the system
 $x^2 + y^2 + 2ax + c^2 = 0$ where c is a constant and a is a variable. [3]
 (b) Solve the differential equation:
 $(x^2 - 2xy + 3y^2) dx + (y^2 + 6xy - x^2) dy = 0$. [7]
 (c) Find the orthogonal trajectory of the family of the cardioids
 $r = a(1 + \cos \theta)$ [6]
5. (a) Solve the differential equation: $(D^2 - 5D + 6)y = e^x \sin x$.
 (b) Solve the differential equation: $y''' + 2y'' - y' - 2y = 1 - 4x^3$. [8+8]
6. (a) Find the Laplace Transformation of the following function
 $e^{-3t}(2\cos 5t - 3\sin 5t)$ [5]
 (b) State and prove convolution theorem to find the inverse of Laplace transforms. [5]
 (c) Use convolution theorem to find
 $L^{-1} \left[\frac{16}{(s^2+4)(s^2+4)} \right]$ [6]
7. (a) Evaluate $\nabla \cdot [r \nabla (1/r^3)]$ where $r = \sqrt{x^2 + y^2 + z^2}$
 (b) Evaluate $\iint_s \mathbf{A} \cdot \mathbf{n} \, ds$ where $\mathbf{A} = 18z\mathbf{i} - 12y\mathbf{j} + 3y\mathbf{k}$ and s is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. [8+8]
8. Verify Stokes theorem for the function $F = x^2\mathbf{i} + xy\mathbf{j}$ integrated round the square whose sides are $x = 0$, $y = 0$, $x = a$ and $y = a$ in the plane $z = 0$. [16]
