Code No: R5210201 **R5**

II B.Tech I Semester(R05) Supplementary Examinations, May/June 2010
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Computer Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Show that β(m,n)=2 $\int\limits_o^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$ and deduce that $\int\limits_o^{\pi/2} \sin^n\theta \, d\theta = \int\limits_o^{\pi/2} \cos^n\theta \, d\theta = \frac{\frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}$.
   (b) Prove that Γ(n) Γ((1-n)= $\frac{\pi}{\sin n\pi}$.
   (c) Show that $\int\limits_0^{\infty} x^m e^{-a^2x^2} dx = \frac{1}{2a^{m+1}}\Gamma\left(\frac{m+1}{2}\right)$ and hence deduce that $\int\limits_o^{\infty} \cos(x^2)dx = \int\limits_o^{\infty} \sin(x^2)dx = 1/2\sqrt{\pi/2}$. [5+5+6]

2. (a) Prove that (2n+1)(1–x^2) P'_n (x)=n(n+1)[P$_{n+1}$(x)-P$_{n-1}$(x)].
   (b) Prove that (1-x^2) P$'_n$(x)=(n+1)[x P$_n$(x)–P$_{n+1}$(x)].
   (c) Show that $\frac{n}{x}J_n(x) + J'_n(x) = J_{n-1}(x)$. [6+5+5]

3. (a) Show that the real and imaginary parts of an analytic function f(z) = u(r,θ) + i v(r,θ) satisfy the Laplace equation in polar form $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ and $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$ respectively.
   (b) If u is a harmonic function, show that w = u^2 is not a harmonic function unless u is a constant. [8+8]

4. (a) Evaluate $\int\limits_c \frac{(z^2-2z-2)}{(z^2+1)^2 z} dz$ where c is $|\, z - i \,| = {}^1\!/\!_2$ using Cauchy's integral formula.
   (b) Evaluate $\int\limits_C (z^2 + 3z + 2)$ dz where C is the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between the points (0,0) to (πa, 2a). [8+8]

5. (a) State and prove Taylor's theorem.
   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3< |z+2| <5. [8+8]

6. (a) Find the poles and residue at each pole of the function $\frac{2z+1}{(1-z^4)}$.
   (b) Evaluate $\int\limits_C \frac{\sin z}{z \cos z} dz$ where C is $|\, z \,| = \pi$ by residue theorem. [8+8]

7. (a) Show that $\int\limits_0^{2\pi} \frac{d\theta}{a+bsin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, a > b> 0 using residue theorem.
   (b) Evaluate by contour integration $\int\limits_0^{\infty} \frac{dx}{(1+x^2)}$. [8+8]

8. (a) Under the transformation w=1/z, find the image of the circle |z-2i|=2.
   (b) Under the transformation $w = \frac{z-i}{1-iz}$, find the image of the circle |z|=1 in the w-plane. [8+8]

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