

II B.Tech I Semester(R05) Supplementary Examinations, May/June 2010

PROBABILITY THEORY AND STOCHASTIC PROCESS

(Common to Electronics & Communication Engineering and Electronics & Computer Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Define probability based on set theory and fundamental axioms.
(b) When two dice are thrown, find the probability of getting the sums of 10 or 11. [8+8]
2. (a) Define and give the concept of random variable.
(b) Define conditional distribution and density functions and explain their properties. [6+10]
3. (a) State and prove properties of variance of a random variable
(b) Let X be a random variable defined by the density function

$$f_X(x) = \begin{cases} \frac{\pi}{16} \cos(\pi x/8) & -4 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$
Find $E[3X]$ and $E[X^2]$. [8+8]
4. (a) State and prove the properties of conditional density functions.
(b) Find the probability density function of $W = X + Y$ where the densities of X and Y are given by
 $f_X(x) = u(x+1) - u(x)$
 $f_Y(y) = \frac{1}{2}[u(y+1) - u(y-1)]$. [8+8]
5. 1. Let X be a random variable that has a mean value.
(a) $\bar{X} = 3$ and variance $\sigma_X^2 = 2$. Let another random variable defined by $Y = -6X + 22$.
 - i. Find R_{XY} , Correlation of X and Y.
 - ii. are X and Y orthogonal
 - iii. are X and Y correlated.
- (b) Find the mean value of a sum of N weighted random variables. [5+3+3+5]
6. (a) Define various random processes
(b) Differentiate between stationary & ergodic random processes.
(c) Show that the random process $X(t) = A \cos(\omega_0 t + \theta)$ is wide sense stationary assume A and ω_0 are constants and θ is a uniformly distributed random variable on the interval $(0, 2\pi)$. [5+5+6]
7. (a) If the PSD of $X(t)$ is $S_{xx}(\omega)$. Find the PSD of $\frac{dx(t)}{dt}$
(b) Prove that $S_{xx}(\omega) = S_{xx}(-\omega)$
(c) If $R(\tau) = ae^{-b|\tau|}$. Find the spectral density function, where a and b are constants. [5+5+6]
8. (a) Define the following random processes
 - i. Band Pass
 - ii. Band limited
 - iii. Narrow band. [3×2 = 6]
- (b) A Random process $X(t)$ is applied to a network with impulse response $h(t) = u(t) \exp(-bt)$ where $b > 0$ is a constant. The Cross correlation of $X(t)$ with the output $Y(t)$ is known to have the same form:
 $R_{XY}(\tau) = u(\tau)\tau \exp(-b\tau)$
 - i. Find the Auto correlation of $Y(t)$
 - ii. What is the average power in $Y(t)$. [6+4]
