Code No: R7210101 **R7**

II B.Tech I Semester(R07) Supplementary Examinations, May/June 2010
MATHEMATICS-II
(Common to Civil Engineering and Bio-Technology)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the rank of the matrix $A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$ by reducing the matrix to echelon form.

   (b) Solve completely the system of equations.
   $x_1 + x_2 - 2x_3 + 3x_4 = 0$
   $x_1 - 2x_2 + x_3 - x_4 = 0$
   $4x_1 + x_2 - 5x_3 + 8x_4 = 0$
   $5x_1 - 7x_2 + 2x_3 - x_4 = 0$ [8+8]

2. (a) Show that the sum of eigen values of a square matrix A is trace A and the product of eigen values is det A.

   (b) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find A^2, A^3, A^{-1}, A^{-2} using Cayley-Hamilton theorem. [8+8]

3. Reduce the quadtratic form $x^2 - 4y^2 + 6z^2 + 2xy - 4xz + 2w^2 - 6zw$ into sum of squares. [16]

4. (a) Obtain the fourier series expansion of f(x) = x in $0 < x < L$

   (b) Find the fourier half-range sine series expansion of the function
   $f(x) = \frac{\pi x}{4}$ for $0 \le x \le \frac{\pi}{2}$
   $= \frac{\pi}{4}(\pi - x)$ for $\frac{\pi}{2} < x \le \pi$ [8+8]

5. (a) Derive the partial differential equation by eliminating the constants from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

   (b) solve the PDE : $z^2 = 1 + p^2 + q^2$

   (c) Find the complete integral of $pe^y = qe^x$ [5+6+5]

6. An insulated rod of length l has its ends A and B maintained at 0^o and 100^oc respectively until steady state conditions prevail. If B is suddenly reduced to 0^oc, find the temperature at a distance x from A at time t. [16]

7. (a) Evaluate $F_c[x^{n-1}]$ if 0<n<1. Deduce that $\frac{1}{\sqrt{x}}$ is self reciprocal under fourier cosine transform.

   (b) Find the finite fourier cosine transform of f(x) if f(x) = $\begin{cases} 1 & if \ 0 < x < \frac{\pi}{2} \\ -1 & if \ \frac{\pi}{2} < x < \pi \end{cases}$

8. (a) If $Z\{u_n\} = \overline{u}(z)$, then prove that $Z(u_{n-k}) = Z^{-k}\ \overline{u}(z)$ (k>0)

   (b) If $Z\{u_n\} = \overline{u}(z)$, then prove that $Z(nu_n)$ = - $Z\ \frac{\partial}{\partial z}\{\overline{u(z)}\}$

   (c) Find $Z^{-1}\left\{\frac{z}{(z-1)(z-2)}\right\}$ [5+5+6]

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