Code No: R7210201 **R7**

II B.Tech I Semester(R07) Supplementary Examinations, May/June 2010
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Evaluate
   i. $\int_0^\infty x^6\, e^{-2x}\, dx$
   ii. $\int_0^\infty e^{-4x}\, x^{3/2}\, dx$
   iii. $\int_0^\infty 3^{-4x^2}\, dx$

   (b) Evaluate
   i. $\int_0^{\pi/2} \sin^6\theta\, \cos^7\theta\, d\theta$
   ii. $\int_0^{\pi/2} \sin^{10}\theta\, d\theta.$ [10+6]

2. (a) Show that an analytic function of constant modulus is constant.
   (b) Find the analytic function whose real part $u = e^{2x}[x\cos 2y - y\sin 2y]$ [8+8]

3. (a) Find the principal value of $(2i)^{1/2}$.
   (b) Find the modulus and argument of $\sqrt{i}$
   (c) Prove that
   i. sinh (iz) = i sin z
   ii. cosh (iz) = cos z
   iii. tanh (iz) = i tan z. [5+5+6]

4. (a) Determine F(2), F(4), F(-3i), $F^1(i)$, $F^{11}(-2i)$, if $F(\alpha) = \int_c \frac{5z^2-4z+3}{z-\alpha} dz$ where c is the ellipse $16x^2+9y^2=144$.
   (b) Use Cauchy's integral formula to evaluate $\oint_c \frac{z+4}{z^2+2z+5} dz$ where c is the circle $|z+1| = 1$ [10+6]

5. Find Maclaurin's series by term wise integrating the integrand
   $f(z) = e^{z^2} \int_0^z e^{t^2} dt.$

6. (a) Determine the residues of the function
   $f(z) = \frac{e^z}{z^2+\pi^2}$ at the poles.
   (b) Find the residues of the function $f(z) = \frac{1-e^{2z}}{z^4}$ at the poles. [8+8]

7. State Rouche's theorem. Prove that $z^7 - 5z^3 + 12 = 0$ all the roots of this equation lie between the circles $|z| = 1$ *and* $|z| = 2$ [16]

8. (a) Find the bilinear transformation which maps vertices (1 + i, -i, 2-i) of the triangle T of the z-plane in to the points (0, 1, i) of the w-plane.
   (b) Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the mapping w = cosh z. [8+8]

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