

Code :9ABS302

R9

II B.Tech I Semester(R09) Supplementary Examinations, May 2011
MATHEMATICS-III

(Electrical & Electronics Engineering, Electronics & Instrumentation Engineering,
 Electronics & Control Engineering, Electronics & Communication Engineering, Electronics
 & Computer Engineering)

Time: 3 hours

Max Marks: 70

Answer any FIVE questions
 All questions carry equal marks

1. (a) Define Beta function and prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
 (b) Prove that:
 - i. $j_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$
 - ii. $j_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$
2. (a) State Cauchy Reimann equations. Show that $f(z) = z + 2\bar{z}$ is not analytic anywhere in the complex plane.
 (b) Define Harmonic function. Find the regular function. Whose imaginary part is $e^x \sin y$.
3. (a) Find all values of z which satisfy.
 - i. $e^z = -2$
 - ii. $e^z = 1 + i$
 (b) Find all principal values of $\left(\frac{\sqrt{3}}{2} + \frac{i}{\sqrt{2}}\right)^{(1+i\sqrt{3})}$
4. (a) Integrate $f(z) = x^2 + ixy$ from A(1,1) to B(2,8) along
 - i. The straight line AB.
 - ii. The curve C: $x=t, y=t^3$.
 (b) Evaluate using cauchy's theorem
 $\int \frac{z^3 e^{-z}}{(z-1)^3} dz$
5. (a) Expand $f(z) = \sin z$ in Taylor's series about $z = \frac{\pi}{4}$
 (b) State Laurent's theorem, find the Laurent series expansion of the function
 $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$
6. (a) Find the residue of $\frac{z^2 - 2z}{(z+1)^2(z^2+1)}$
 (b) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$
7. (a) State Rouché's theorem. Use Rouché's theorem to find the number of zero's of the polynomial $z^{10} - 6z^7 + 3z^3 + 1$ if $|z| < 1$
 (b) Show that the equation $z^4 + 4(1+i)z + 1 = 0$ has one root in each quadrant.
8. (a) Show that the transformation $w = \frac{1}{z}$ maps a circle to a circle or to a straight line if the former goes through the origin.
 (b) Find the bilinear transformation which maps $\infty, i, 0$ in the z -plane in to $-1, -i, 1$ in the w -plane.
