Code: R5100102

R05

B. Tech I Year (R05) Supplementary Examinations, May 2012
MATHEMATICS - I
(Common)

Time: 3 hours | Max Marks: 80

Answer any FIVE questions
All questions carry equal marks
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1 (a) Test the convergence of following series:

(i) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ (ii) $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n-1}\right)$

(b) Discuss the convergence of series: $\frac{1}{2}x + \frac{1.2}{2.5}x^2 + \frac{1.2.3}{2.5.8}x^3 + \ldots\ldots\ldots$(x>0).

2 (a) A rectangular box open at the top is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

(b) Find the circle of curvature of the curve at (0, 1) on the curve $y = x^3+2x^2+x+1$.

3 (a) Trace the curve $Y^2(a^2+x^2) = x^2(a^2-x^2)$, a>0.

(b) Find the length of the arc of the curve $x = e^{\theta}\sin\theta$, $y = e^{\theta}\cos\theta$ from $\theta = 0$ to $\frac{\pi}{2}$.

4 (a) Solve $x\frac{dy}{dx} + y = logx^2$.

(b) Solve $\frac{d^2y}{dx^2} + a^2y =$ secax.

5 (a) Evaluate $\iint y\,dx\,dy$ in the region R bounded by y –axis, the curve $y = x^2$ and the line x+y = 2 in the first quadrant.

(b) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x}$ log z d z dxdy.

6 (a) Find: (i) L {cos(at+b)} (ii) L{e^{-2t}(t-5)u(t-5)}

(iii) $L^{-1}\left\{\log\left(\frac{s+3}{s+4}\right)\right\}$ (iv) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

(b) Using Laplace transform, value $(D^2+5D-6)y=x^2e^{-x}$, y(0) = a, $y^1(0)$ = b.

7 (a) Find the directional derivative of f(x, y, z) = zx^2-xyz at the point (1, 3, 1) in the direction of the vector $3\bar{i}-2\bar{j}+\bar{k}$.

(b) Evaluate $\int_c \bar{F}.d\bar{x}$ where $\bar{F}$ = (x-3y) $\bar{i}$+(y-2x) $\bar{j}$ and C is the closed curve in the xy-plane x= 2 cost y = 3 sint from t = 0 to 2 π.

8 (a) Verify Green's theorem $\int_c\left[\left(3x-8y^2\right)dx+\left(4y-6xy\right)dy\right]$ where C is the boundary of the region bounded by x=0, y=0 and x+y = 1.

(b) Apply Stoke's theorem to evaluate $\int_c$ (ydx+zdy+xdz), where C is the curve of intersection of $x^2+y^2+z^2 = a^2$ and x+z = a.

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