Code: 9ABS104

1

B. Tech I Year (R09) Regular & Supplementary Examinations, May 2012

MATHEMATICS - I

(Common to all Branches)

Time: 3 hours Max Marks: 70

Answer any FIVE questions

All questions carry equal marks

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1 (a) Solve : $(y^2 - 2xy)dx = (x^2 - 2xy)dy$.

(b) Solve : $(x^2 - ay)dx = (ax - y^2)dy$.

2 (a) Solve the differential equation: $(1+x)^2 \frac{d^3y}{dx^3} + (1+x)\frac{dy}{dx} + y = 4\cos\ \log(1+x)$.

(b) Solve the differential equation $(D^3 - 1)\, y = e^x + \sin^3 x + 2$.

3 (a) Verify Rolle's theorem for $f(x) = x^2 - 5x + 6$ in [2, 3].

(b) Examine if Rolle's theorem is applicable for the function $f(x) = \tan x$ in $[0,\pi]$.

4 (a) Find the surface area of the right circular cone generated by the revolution of a right angled triangle about a side which contains a right angle.

(b) Find the cost of plating of the front portion of the parabolic reflector of an automobile head light of 12cm diameter and 4cm deep if the cost of plating is Rs.2.00 per sq. cm.

5 (a) Evaluate $\iint r\sin\theta\ dr\, d\theta$ over the cardioids $r = a(1+\cos\theta\,)$ above the initial line.

(b) Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dx\,dy\,dz$.

6 (a) Find the Laplace transform of: (i) $t\sin 3t\cos 2t$ (ii) $t^2e^{-2t}\cos t$.

(b) Apply Convolution theorem to find $L^{-1}\left\{\frac{1}{s^2(s+a)^2}\right\}$.

7 (a) Solve the D.E $\frac{dy}{dt} + 4y + 5\int_0^t y dt = e^{-t}, y(0) = 0$. Using Laplace transform.

(b) Using Laplace transform, Evaluate $\int_0^\infty t^3e^{-t}\sin t\ dt$.

8 State Stoke's theorem and verify Stoke's theorem for $\overline{F} = (x^2+y^2)\boldsymbol{i} - 2xy\,\boldsymbol{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = \pm b$.

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B. Tech I Year (R09) Regular & Supplementary Examinations, May 2012
MATHEMATICS - I
(Common to all Branches)

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1 (a) A body initially at 80^0 C cools down to 60^0 C in 20 min. The temperature of the air is 40^0 C. Find the temperature of the body after 40 min.
(b) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after 1 ½ hours.

2 (a) Solve: $(D^2 + D + 1)\, y = x^3$.
(b) Solve: $(D^2 – 3D + 2)\, y = 2x^2$.

3 (a) Find the minimum value of $x^2 + y^2 + z^2$ given that $xyz = a^3$.
(b) Examine the function for extreme values: $f(x, y) = x^4 + y^4 – 2x^2 + 4xy – 2y^2$ $(x > 0, y> 0)$.

4 (a) Find the surface of the solid generated by the revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.
(b) Find the area of the surface of revolution formed by revolving the loop of the curve $9\, ay^2 = x\, (3a – x)^2$ about the x-axis.

5 (a) Find $\iint (x+y)^2\; dx\, dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
(b) Change the order of integration in the integral $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} f(x,y)dx\, dy$.

6 (a) If $L\{f(t)\} = F(s)$ *then prove that* $L\left\{\int_0^t f(t)dt\right\} = \frac{F(s)}{s}$.
(b) Find $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s^2}\right)\right\}$.

7 (a) Using Laplace transform, show that $\int_0^\infty t^2\, e^{-4t} \sin 2t\, dt = \frac{11}{500}$.
(b) Solve the D.E $y'' + n^2 y = a\sin(nt+2)\, , y(0) = 0, y'(0) = 0$ Using Laplace transform.

8 State Stoke's theorem and verify Stoke's theorem for a vector field defined:
$\overline{F} = -y^3 \boldsymbol{i} + \boldsymbol{x^3}\, \boldsymbol{j}$, in the region $x^2 + y^2 \leq 1\, , z = 0$.

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3

B. Tech I Year (R09) Regular & Supplementary Examinations, May 2012
MATHEMATICS - I
(Common to all Branches)

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1 (a) Solve the differential equation (2x – y + 1) dx + (2y – x – 1) dy = 0.
(b) Solve: (hx + by + f)dy + (ax + hy + g) dx = 0.

2 (a) Solve: $(D^2 + 4)\, y = e^x + \sin 2x + \cos 2x$.
(b) Solve: $(D^2 - 4D + 3)\, y = \sin 3x \sin 2x$.

3 (a) Find the shortest distance from origin to the surface $xyz^2 = 2$.
(b) Investigate for the maxima and minima, if any of $x^3y^2 (1 - x - y)$.

4 (a) Trace the curve: $x^3 + y^3 = 3axy$.
(b) Trace the curve: $(a^2 + x^2)y = a^2x$.

5 (a) Evaluate $\int_0^1 \int_0^{x^2} e^{y/x}\, dy\, dx$.
(b) Change the order of integration and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$.

6 (a) If $f(t)$ is a periodic function with period T, prove that $L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt$.
(b) Use Heaviside's expansion formula to find $L^{-1}\left\{\frac{1}{s^3+1}\right\}$.

7 (a) Find $L\{\sin\sqrt{t}\}$ *and Hence Evaluate* $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$.
(b) Solve the D.E. $y'' + 4y' + 3y = e^{-t}$, $y(0) = 1,\ y'(0) = 1$. Using L.T.

8 (a) If $r = x\bar{i} + y\bar{j} + z\bar{k}$, show that $\nabla r^n = nr^{n-2}\bar{r}$.
(b) Find the works done in moving in a particle in the force field $\bar{F} = (3x^2)\boldsymbol{i} + (2zx - y)\,\boldsymbol{j} + z\boldsymbol{k}$**,** along (i) the straight line form (0,0,0) to (2,1,3) **(**ii) the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from x= 0 to x= 2.

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1 (a) Find the equation of the system of orthogonal trajectories of the family of curves $r^n \sin n\theta = a^n$ where a is the parameter.

(b) Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$.

2 (a) Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$.

(b) Solve: $(D^3 - 5D^2 + 8D - 4)\, y = e^{2x}$.

3 (a) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ up to the terms of 3rd degree.

(b) Expand $x^3 + y^3 + xy^2$ in powers of $(x - 1)$ and $(y - 2)$ using Taylor's series.

4 (a) Find the radius of curvature at the origin of the curve $y^2 = \frac{x^2(a+x)}{(a-x)}$.

(b) Find the radius of curvature at the origin for the curve $y^4 + x^3 + a(x^2 + y^2) - a^2y = 0$.

5 (a) Evaluate $\int_0^3 \int_1^2 xy(1 + x + y)\, dy\, dx$.

(b) Evaluate the integral by changing the order of integration $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)\ dxdy$.

6 (a) Find the Laplace transform of: (i) $e^{-3t}(2\cos 5t - 3\sin 5t)$. (ii) $e^{3t}\sin^2 t$.

(b) Find $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$ Using Convolution theorem.

7 (a) Solve the D.E. $y'' + 2y' - 3y = \sin t$, $y(0) = 0$, $y'(0) = 0$. Using Laplace transform.

(b) Using Laplace transform, Evaluate $\int_0^\infty e^{-4t}\frac{(2\sin t - 3\sinh t)}{t}\, dt$.

8 State Stoke's theorem and verify Stoke's theorem for a vector field defined: $\overline{F} = (2x - y)\boldsymbol{i} - yz^2\,\boldsymbol{j} - y^2z\,\boldsymbol{k}$, over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy –plane.

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