Code: 9ABS302 **R09**

II B.Tech I Semester (R09) Supplementary May 2012 Examinations
MATHEMATICS-III

(Common to Electrical & Electronics Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Communication Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max. Marks: 70**

Answer any FIVE questions
All questions carry equal marks

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1. (a) Show that $\beta(m, n) = \Gamma(m)\Gamma(n)/\Gamma(m+n)$
   (b) Show that $\int_0^1 \frac{x^n}{\sqrt{1-x^2}} dx = \frac{2.4.6.........(n-1)}{1.3.5..........n}$
   (c) Show that $\int_0^{\pi/2} \sqrt{\tan\theta}\ d\theta = 1/2\Gamma(1/4)\ \Gamma(3/4)$

2. (a) Prove that
   (a) $P_{n+1}^1(x) - P_{n-1}^1(x) = (2n+1)P_n(x)$.
   (b) $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$
   (b) When n is an integer? Show that $J_n(x) = (-1)^n J_n(x)$.

3. (a) Find the analytic function whose imaginary part is $f(x, y) = x^3y - xy^3 + xy + x + y$ where $z = x + iy$.
   (b) Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\mathrm{Re}\ f(z)|^2 = 2|f^1(z)|^2$ where f|z| is analytic.

4. (a) Evaluate $\int_c \frac{(z^3 - \sin 3z)}{(z-\pi/2)^3} dz$ with c:$|z| = 2$ using Cauchy's integral formula.
   (b) Evaluate $\int_{0,0}^{1,1} (3x^2 + 4xy + ix^2)dz$ along $y = x^2$.
   (c) Evaluate $\int_c \frac{dz}{e^z(z-1)^3}$ where c: $|z| = 2$ using Cauchy's integral theorem.

5. (a) State and prove Laurent's theorem.
   (b) Obtain all the Laurent series of the function $\frac{7z-2}{(z+1)z(z+2)}$ about $z = -2$.

6. (a) Find the poles and the residue at each pole of $f(z) = \frac{z}{z^2+1}$.
   (b) Evaluate $\int_c \frac{ze^z dz}{(z^2+9)}$ where c is $|z| = 5$, by residue theorem.

7. (a) Show that $\int_0^{2\pi} \frac{d\theta}{a + b\sin\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ $(a > b > 0)$ using residue theorem.
   (b) Evaluate by contour integration $\int_0^{\infty} \frac{dx}{1+x^2}$.

8. (a) Find the image of the infinite strip $0 < y < 1/2$ under the transformation $w = 1/z$.
   (b) Find the bilinear transformation which maps the points (-1, 0, 1) into the points (0,i,3i).

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