II B.Tech II Semester (R09) Regular & Supplementary April/May 2012 Examinations

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electronics & Computer Engineering)

Time: 3 hours **Max Marks: 70**

Answer any FIVE Questions

All Questions carry equal marks

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1. (a) Discuss the probability as a relative frequency with an example.
   (b) Using Venn diagrams for three sets A,B,C shade the areas corresponding to the sets.
   (a) $(A \cup B) - C$ (b) $\overline{B} \cap A$ (c) $A \cap B \cap C$ (d) $\overline{A \cup B} \cap C$
   (c) State and prove total probability theorem.

2. (a) Define a random variable. When a random variable is said to be Gaussian? Explain.
   (b) Consider the probability density $f_x(x) = a.e^{-b|x|}$, where x is random variable whose allowable values ranges from $x = -\infty$ to $+\infty$. Find :(a) The CDF (b) The relation between a and b .(c) The probability that x lies between a and b. (c) The probability that x lies between 1 and 2.

3. (a) State and prove the properties of moment generating function of a random variable.
   (b) Find the mean and variance of the Gaussian random variable.

4. (a) State and explain central limit theorem.
   (b) Let X and Y be two standardized Gaussian Random variables. Find the density of z=x+y.

5. (a) Find the joint moment generating function of two normally distributed random variables X and Y.
   (b) Let X,Y, and Z be independent zero mean, unit variance Gaussian random variables. Find the pdf of R=$\sqrt{x^2 + y^2 + z^2}$.

6. (a) Explain, how a process is called stationary to order two, with an examples?
   (b) A random process defined as $x(t) = A.\sin(wt + \theta)$ where 'A' is a constant and 'θ' is a random variable, uniformly distributed over($-\pi, \pi$). Check $x(t)$ for stationarity.

7. (a) Explain ergodic random process.
   (b) A random process is given by
   $$x(t) = A \sin(w_0 t + \theta)$$
   Where A and w_0 are constants and θ is a random variable uniformly distributed over the interval ($-\pi, \pi$) consider a new random process Y(t)= X^2(t).
   (i) Find the auto correlation function of Y(t).
   (ii) Find the cross correction function of X(t) and Y (t)

8. (a) Derive the relationship between cross-power spectrum and cross-correlation function of random processes.
   (b) A random process defined by
   $Y(t) = X(t) - X(t-a)$ where $X(t)$ is a wide sense stationary process and a>o is a constant. Find the PSD of $Y(t)$ in terms of the corresponding quantites of $X(t)$.

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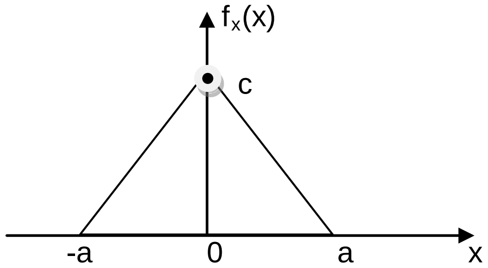
1. (a) Discuss following set operations with an example
   (i) Complement (ii) Algebra of sets (iii) De-Morgan's law (iv) Duality principle.
   (b) Use Venn diagrams to prove De-Morgan's laws $\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cap \overline{B}$.
   (c) State and prove total probability theorem.

2. (a) What are the methods of defining conditioning event? Define the conditional denisty function and explain the significance of its properties.
   (b) The random variable x has the pdf shown below



(a)Find $f_x(x)$, (b) Find the CDF of X (c) Find b such that P $\{|X| < b\}$ ½.

3. (a) State and explain Chebychev's inequality.
   (b) A random variable x has the following density

$$f_x(x) = \begin{cases} \left(\frac{3}{32}\right)(-x^2 + x - 12 & 2 \le x \le 6 \\ 0 & \text{else where} \end{cases}$$

Find the following moments:
   (i) m_0 (ii) m_1 (iii)m_2

4. (a) What is point conditioning? Discuss with an example.
   (b) Let X,Y,Z have the joint pdf

$$f_{x,y,z}(x, y, z) = k(x + y + z) \quad 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$$

   (i) Find K (ii) Find $f_z(^z/_{x}, y)$

5. What is linear transformation? Explain in terms of Gaussian random variable.

6. (a) Define random process. Discuss the classification in detail.
   (b) A random process $X(t)$ is defined as $X(t) = (A + 1)\cos t + B\sin t$ where A nd B are independent random variables with zero mean and same mean square of 1. Verify that $X(t)$ is not stationary,but covariance stationary.

7. (a) Prove that $|R_{xx}(\tau)| \le \sqrt{[R_{xx}(0).R_{yy}(0)]}$
   (b) Assume that an ergodic random process x(t) has an auto correlation function

$$R_{xx}(\tau) = 18 + \frac{2}{-6 + \tau^2}[1 + 4\cos(12\tau)]$$

   (i) Find $[\overline{X}]$
   (ii) Does this process have a periodic component . Find the period.

8. (a) Determine the relationship between power spectrum and auto correlation function.
   (b) Consider a train of rectangular pulses having an amplitude of 2 volts and width which are either 1μsec or 2 μ sec with equal probability. The mean time between the pulses is 5 μsec. Find the PSD of the pulse train.

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1. (a) Write about the mathematical model of experiments with an example.
   (b) In a box there are 500 colored balls:75 black, 150 green, 175 red, 70 white, and 30 blue. What are the probabilities of selecting a ball of each color?
   (c) State and prove Baye's theorem.

2. (a) Explain conditional distribution and state its properties.
   (b) Consider the random experiment of rolling two dies simultaneously. Let x_i be the outcome of first die and y_j be the outcome of second die. In other words 'i' is the number shown by the first die and 'j' be the number shown by the seocnd die. Explain and plot CDF for the sum of the number shown by the two dies.

3. (a) Discuss the application of the moment generating function with an example.
   (b) Compare the Chebysher bound and the exact probability for the event $\{|x - m| \geq c\}$ as a funciton of c for
      (i) X a uniform random variable in internal [-b, b]
      (ii) X a zero mean Gaussian random variable.

4. The joint probability density of a function is given by

$$f_{x,y}(x,y) = \begin{cases} 1/ab & 0 < x < a \ \ and \ 0 < y < b \\ 0 & \text{else where} \end{cases}$$

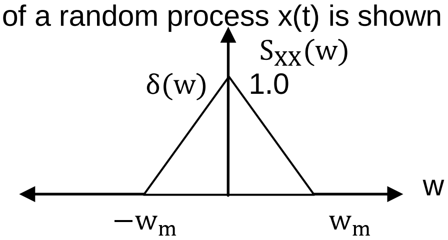
   (i) Find and sketch $f_{x,y}(x,y)$.
   (ii) If a<b, find (a) P{x+y ≤3a/4} (b) P{Y≤2bx/a}

5. (a) Write short notes an W-variants Gaussian density function.
   (b) If the sample mean and variance are given by
   $\mu=\frac{X_1+X_2}{2}$ and $V=\frac{(X_1-\mu)^2+(X_2-\mu)^2}{2}$
   Find the joint pdf in terms of x_1 and x_2.
   Evaluate the joint pdf if the x_1 and x_2 are independent exponential random variables with the same parameter.

6. (a) Define random process. Discuss the classification in detail.
   (b) A random process is defined as $X(t) = A.\sin(wt + \theta)$ where A is constant and 'θ' is a random variable uniformly distributed over (-π, π). Check $X(t)$ for stationary.

7. (a) State and explain the properties of poise random process.
   (b) A Gaussian random process has an autocorrelation function $R_{xx}(\tau) = 6\exp(-|\tau|/2)$. Determine a co-variance matrix for the random variables x(t), x(t+1), x(t+2) and x(t+3).

8. (a) Derive the relationship between PSDs of input and output random process of an LTI system.
   (b) The PSP of a random process x(t) is shown below



   (i)Find the auto-correlation function $R_{xx}(\tau)$ of x(t) (ii) What is the mean square value of x(t)?

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1. (a) Discuss the following set operations with as example.
   (i) Venn diagram (ii) Equality and difference (iii) Union and intersection (iv) complement.
   (b) Two set are given by A={-6,-4,-0.5,0.1,6,8} and B={-0.5,0.1,2,4}. Find:
   (i) A-B (ii) A∪B (iii) A∩B.
   (c) State and prove Baye's theorem.

2. (a) Explain the following:
   (a) Uniform random variable (ii) Exponential random variable.
   (b) The CDF of the random variable x is given by
   $$f_x(x) = \begin{cases} (1/3) + (2/3)(x+1)^2 & -1 \le x \le 0 \\ 0, & x \le -1 \end{cases}$$
   Find the probability of the events A={x>1/3},
   B={$|x| \ge 1$}, C={$|x - 1/3| \le$} , D= {x<0}.

3. (a) Discuss the transformation of a random variable.
   (b) Find the mean and variance of the discrete uniform random variable that takes on for the set {1,2,……..n} with equal probability.

4. A joint sample space for two random variables X and Y has four elements (1,1),(2,2),(3,3) and (4,4). Probability of these elements are 0.1,0.35,0.05,and 0.5 respectively.
   (i) Determine through logic and sketch the distribution function $F_{x,y}(x,y)$.
   (ii) Find the probability of the event {x≤2.5;y≤ 6}.
   (iii) Find the probability of the event {x≤3}.
   (iv) Sketch the marginal distribution functions.

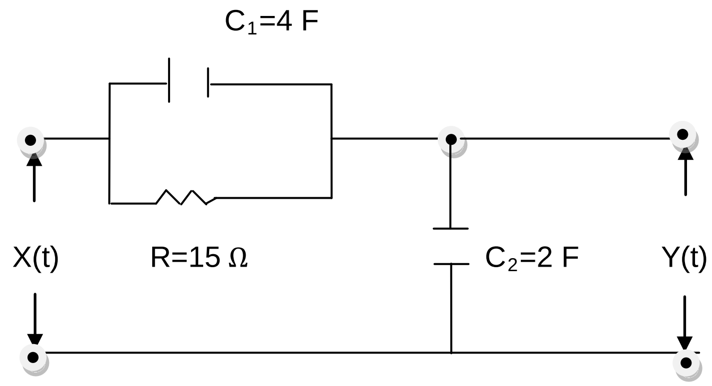
5. (a) Show that the variance of a weighted sum of uncorrected random variables equals the weighted sum of the variance of the random variables.
   (b) If x and y are independent exponential random variables with parameter ∝=1. Find:
   (i) E[X+Y] (ii) E[X-Y].

6. (a) Discuss the statistical independence of two processes with an example.
   (b) Sample functions in a discrete random process are constants ;that is X(t)=C=constant. Where C is a discrete random variable having possible values C_1=1, C_2=2and C_3=3 occurring with probabilities 0.6,0.3 and 0.1 respectively.
   (i) Is X(t) deterministic.
   (ii) Find the first-order density function of X(t) at any time 't'.

7. (a) State and prove the properties of auto-correlation function.
   (b) Statistically independent, zero-mean random processes X(t) and Y(t) have auto-correlation function
   $R_{XX}(\tau)=e^{-|\tau|}$ and $R_{YY}(\tau)=\cos(2\pi\tau)$ respectively
   (i) Find the auto-correlation function of the sum
   $W_1(t)$=X(t)+Y(t).
   (ii) Find the auto-correlation function of the difference
   (iii) $W_2(t)$=X(t)-Y(t).

Contd. in Page 2

8. (a) Discuss the cross correlation between the input x(t) and output Y(t) of an LTI system.
   (b) A stationary random process X(t) having a auto-correlation function $R_{XX}(\tau) = 2.e^{-u|\tau|}$ is applied to a network shwon below:



Find (a) $S_{XX}(w)$ (b) $S_{YY}(w)$

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